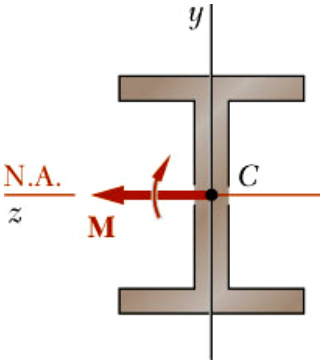
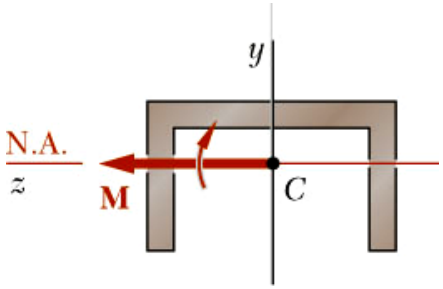
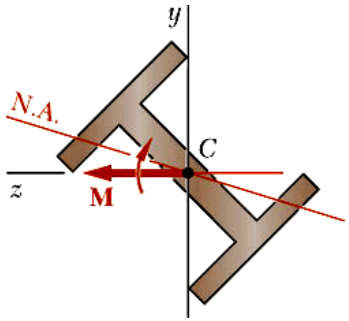
Simetrik Olmayan Eğilme (Eğik Eğilme)

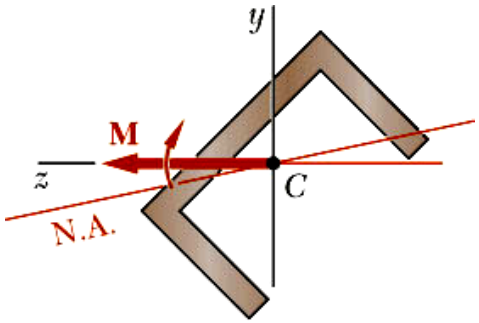


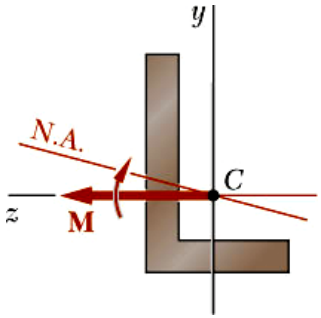


Simetrik Eğilme :

- Eğilme momenti bir simetri düzleminde (*y* burada simetri eksenidir) etki etmektedir.
- Kesit eğilmeden sonra yine simetrik kalır ve eğilme simetri düzlemindedir.
- Tarafsız eksen ile moment ekseni (yönü) çakışıktır.







Simetrik Olmayan Eğilme :

- Kesitte simetri düzlemleri veya eksenleri olmasına rağmen; *eğilme momenti bir simetri düzleminde etki etmeyebilir.*
- Eğilme, momentin etki düzleminde gerçekleşmez.
- Genel olarak, tarafsız eksenle moment ekseni (yönü) çakışık değildir.

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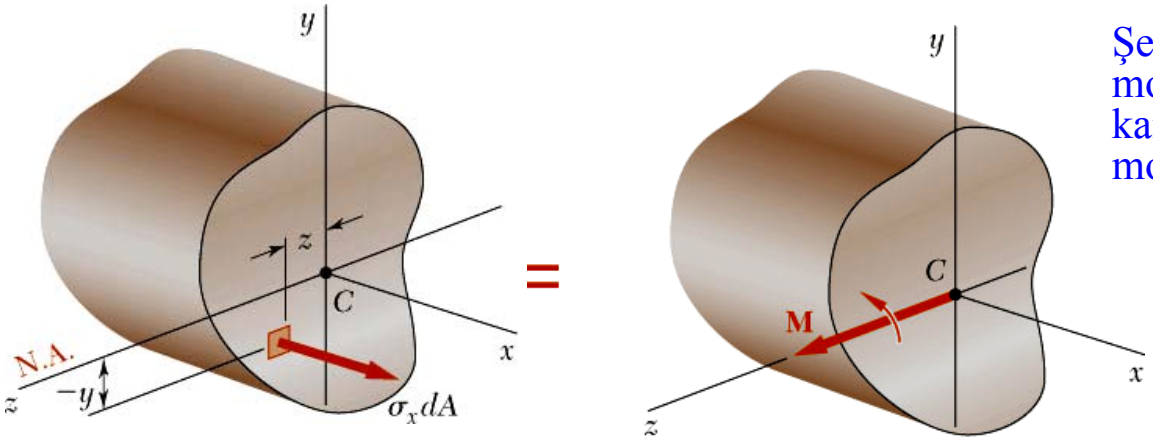
- Keyfi bir kesitte tarafsız eksen ile moment ekseni hangi şartlarda çakışıktır?



Şekildeki gibi bir *Mz* eğilme momentine maruz kesitteki gerilmelere karşılık gelen eşdeğer kuvvet ve momentler şu şartları sağlamalıdır:

$$F_x = \int \sigma_x \, dA = 0$$
$$M_y = \int z\sigma_x \, dA = 0$$
$$M_z = \int -y\sigma_x \, dA = M$$

$$F_x = \int \sigma_x dA = \int \left(-\frac{y}{c}\sigma_m \right) dA = 0$$

veya $\int y \, dA = 0$

Tarafsız eksen geometrik merkezden geçer

$$M = M_z = -\int y \left(-\frac{y}{c}\sigma_m \right) dA = \frac{\sigma_m I}{c}$$

$$\sigma_m = \frac{M_z c}{I_z}$$ *normal gerilme ifadesi*

$$M_y = \int z\sigma_x dA = \int z \left(-\frac{y}{c}\sigma_m \right) dA = 0$$

$$\int yz \, dA = 0 = I_{yz}$$

Bunun için moment vektörü geometrik merkezden geçen ***asal atalet eksenlerinden*** *birisiyle çakışmalıdır. Eğer uygulanan moment vektörünün yönü* ***asal atalet eksenlerinden*** *birisiyle çakışmıyorsa* ***eğik eğilme*** *meydana gelir.*

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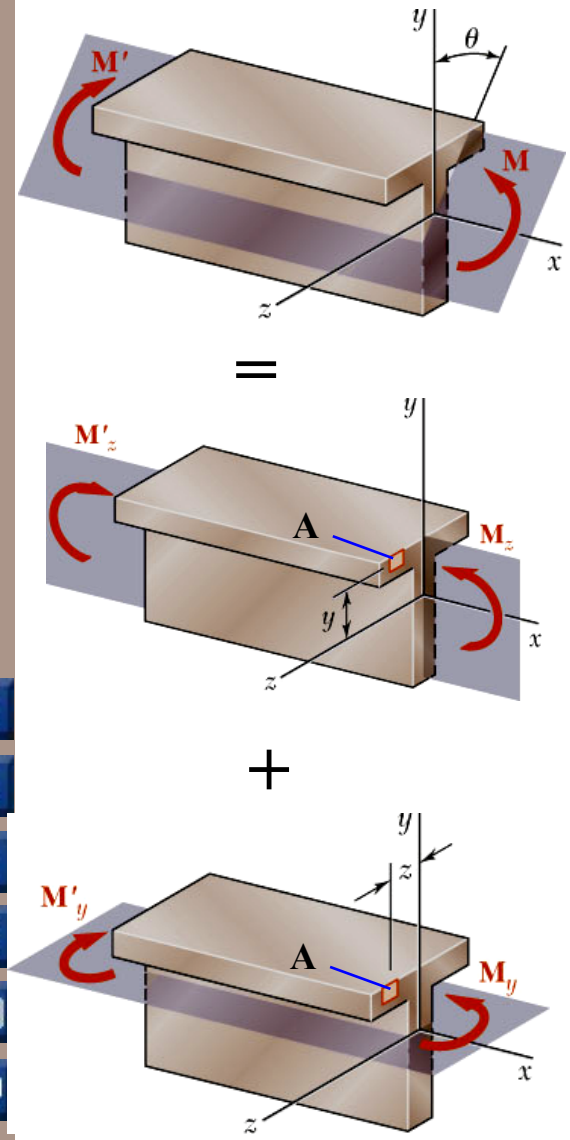
Simetrik olmayan eğilmede süperpozisyon prensibi:



$$M_z = M\cos\theta \qquad M_y = M\sin\theta$$

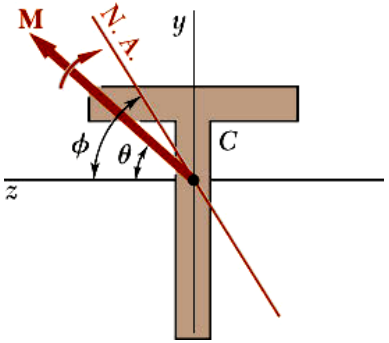
Pozitif bölgede yer alan bir ***A*** noktasındaki bileşke gerilme;

$$\sigma_x = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

- **Tarafsız eksenin konumu için;**

$$\sigma_x = 0 = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y} = -\frac{(M\cos\theta)y}{I_z} + \frac{(M\sin\theta)z}{I_y}$$

$$\tan\phi = \frac{y}{z} = \frac{I_z}{I_y}\tan\theta$$



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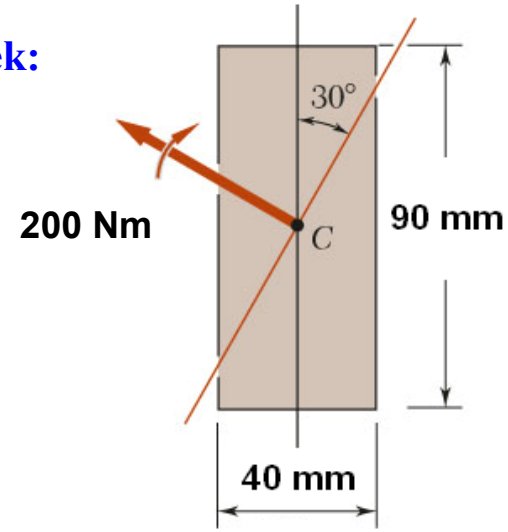
Örnek:



Şekildeki yükleme durumunda;

a) Kirişteki maksimum gerilmeyi,

b) Tarafsız eksenin yatay eksenle yaptığı açıyı bulunuz.

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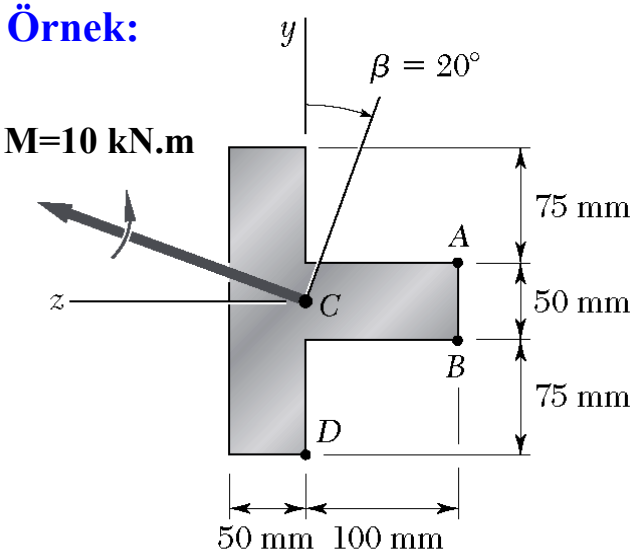
Örnek:



Şekildeki yükleme durumunda;

a) Tarafsız eksenin yatayla yaptığı açıyı bulunuz.

b) A ve D noktalarındaki gerilmeleri bulunuz.

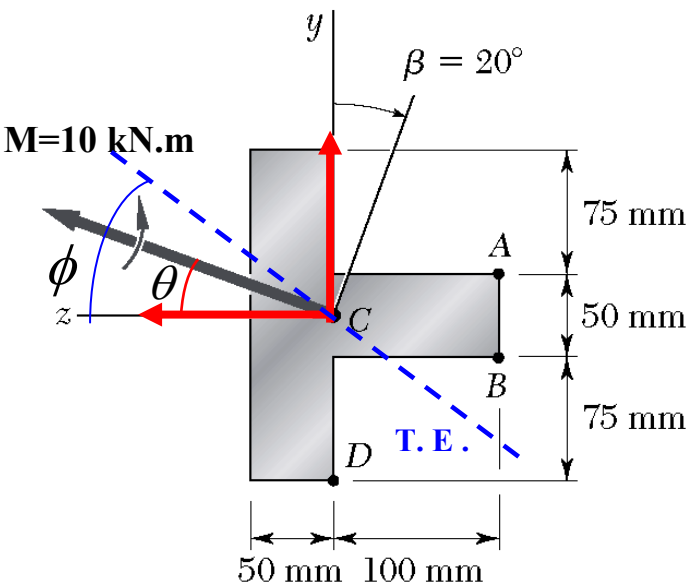
Burada; ***C*** noktası geometrik merkez, ***z*** ekseni simetri ekseni ve ***y*** ile ***z*** asal atalet eksenleridir.

$$I_y = \frac{1}{3}\cdot 200\cdot 50^3 + \frac{1}{3}\cdot 50\cdot 100^3 = 25\times 10^6\, mm^4$$

$$I_z = \frac{1}{12}\cdot 50\cdot 200^3 + \frac{1}{12}\cdot 100\cdot 50^3 = 34,375\times 10^6\, mm^4$$

$$M_y = M\cdot \sin\theta = 10.\sin 20 = 3,42\times 10^6\, N\cdot mm^4$$

$$M_z = M\cdot \cos\theta = 10.\cos 20 = 9,4\times 10^6\, N\cdot mm^4$$



$$tg\phi = \frac{I_z}{I_y} tg\theta = \frac{34,375}{25} tg20 = 0,5 \Rightarrow \phi = 26,6^o$$

$$\sigma_x = -\frac{M_z}{I_z} y + \frac{M_y}{I_y} z$$

$$\sigma_A = -\frac{9,4\times 10^6}{34,375\times 10^6} 25 + \frac{3,42\times 10^6}{25\times 10^6}(-100) = -20,52 MPa$$

$$\sigma_D = -\frac{9,4\times 10^6}{34,375\times 10^6}(-100) + \frac{3,42\times 10^6}{25\times 10^6}(0) = 27,34 MPa$$

D (z,y)=D (0,-100) ; A (z,y)=D (-100,25)

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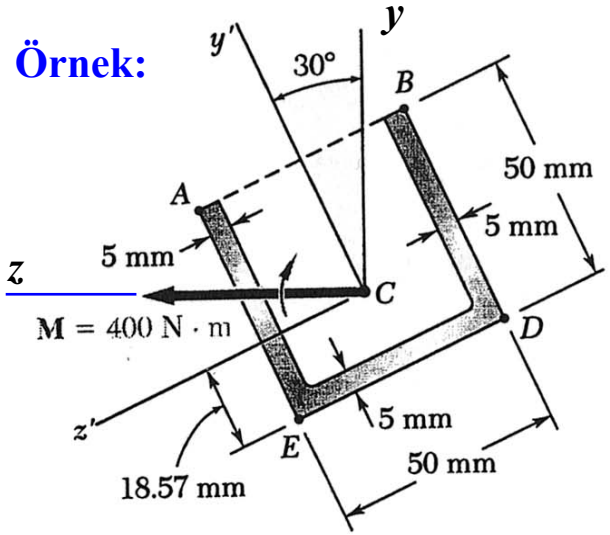
Örnek:



Şekildeki yükleme durumunda;

a) Tarafsız eksenin yatayla yaptığı açıyı bulunuz.

b) En büyük çeki gerilmesini bulunuz.

$$I_{y'} = 281 \times 10^3 \quad mm^4; \quad I_{z'} = 176.9 \times 10^3 \quad mm^4$$

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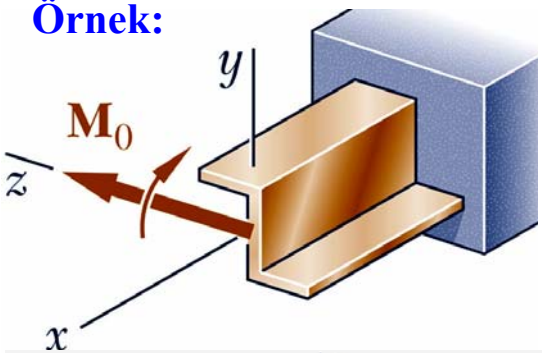
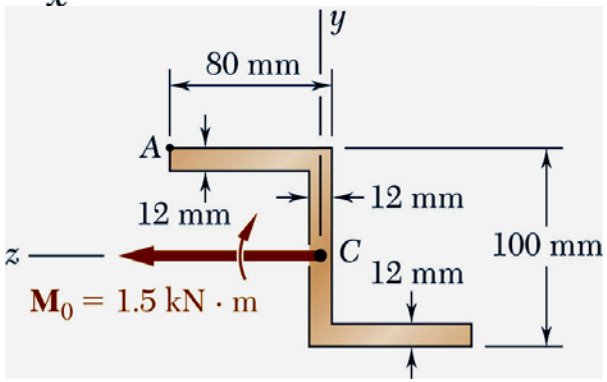
Örnek:



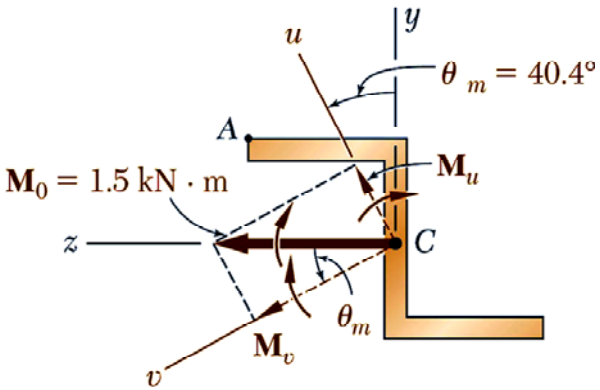
Şekildeki yükleme durumunda;

a) A noktasındaki gerilmeyi,

b) Tarafsız eksenin yatayla yaptığı açıyı bulunuz.



I. YOL : asal atalet eksenleri kullanılarak çözüm yapılırsa;



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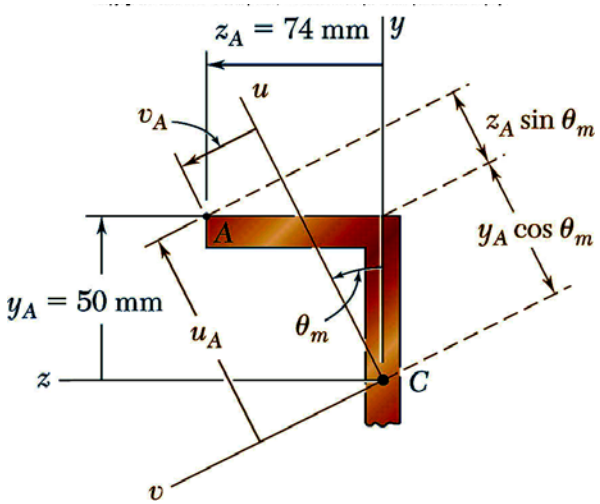

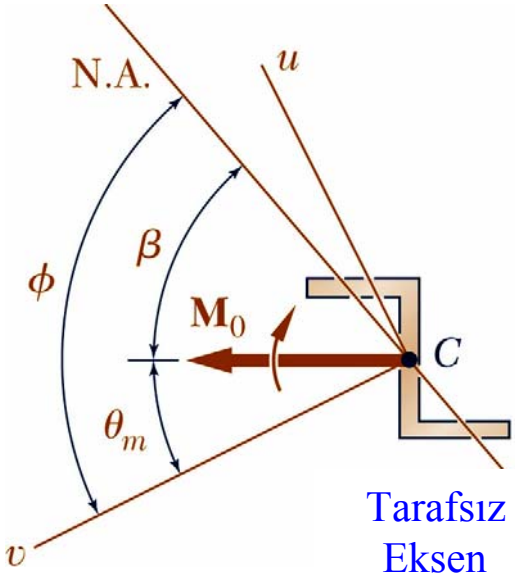

Tarafsız
Eksen

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II. YOL

$$\sigma_x = \frac{M_z\left(I_{yz}\ z - I_y\ y\right) + M_y\left(I_z\ z - I_{yz}\ y\right)}{I_y\ I_z - I_{yz}^2}$$

Bağıntısı ile doğrudan çözüme ulaşılabilir.

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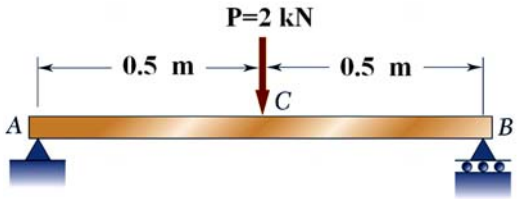
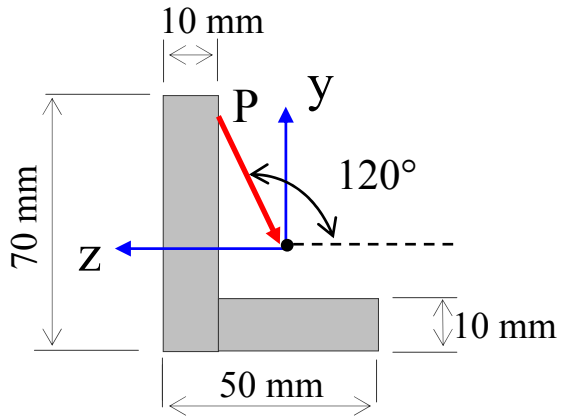
Örnek:





Şekildeki kiriş için;

a) geometrik merkezin koordinatlarını ve I_y , I_z , I_{yz} atalet momentlerini

b) Asal atalet momentlerini ve doğrultularını

c) En büyük çeki ve bası gerilmelerini bulunuz.

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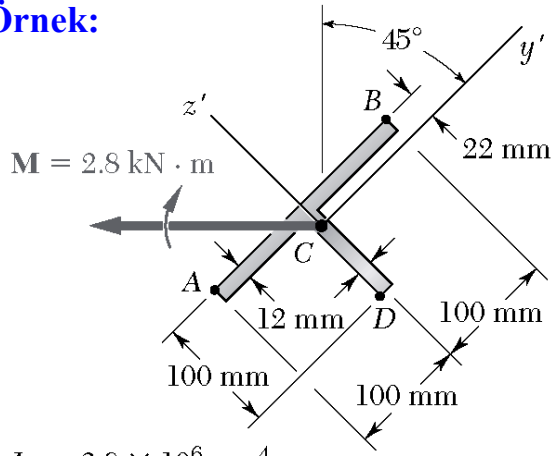
Örnek:



$I_{y'} = 2.8 \times 10^6\ \text{mm}^4$

$I_{z'} = 8.9 \times 10^6\ \text{mm}^4$

Şekildeki yükleme durumunda;

a) Tarafsız eksenin yatayla yaptığı açıyı bulunuz.

b) En büyük çeki gerilmesini bulunuz.

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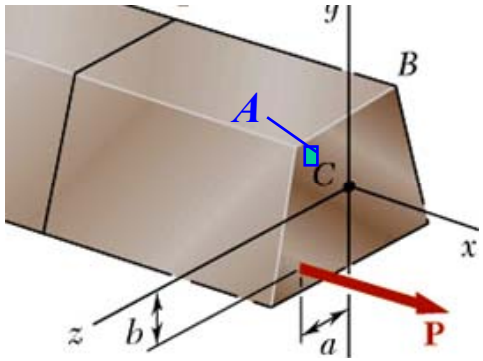
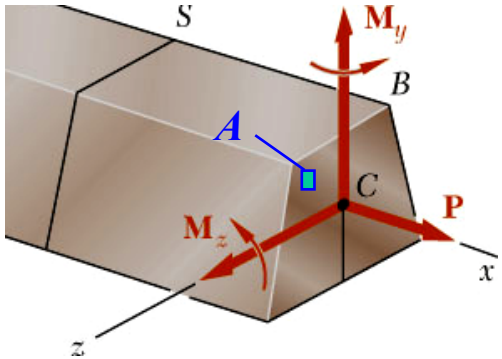
Eksantrik Yüklemede Genel Durum





$P =$ centric force

$$M_y = Pa \qquad M_z = Pb$$

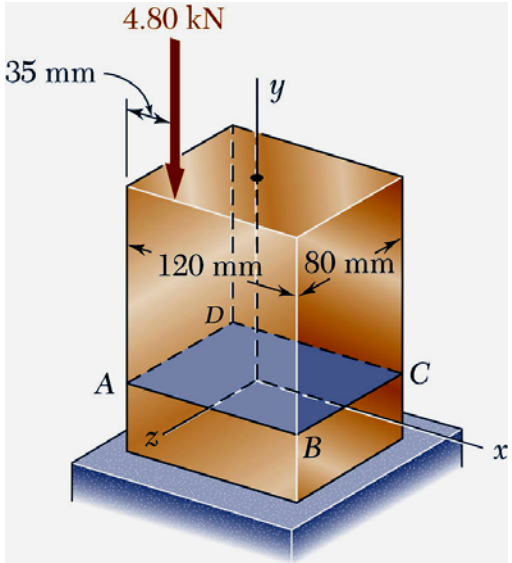
Pozitif bölgede bir *A* noktası için Süperpozisyon prensibi:

$$\sigma_x = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

Tarafsız eksenin konumu:

$$\sigma_x = 0 \longrightarrow \frac{M_z}{I_z} y - \frac{M_y}{I_y} z = \frac{P}{A}$$

Örnek



Şekildeki geometri ve yükleme durumu için

a) A,B,C ve D noktalarındaki gerilmeleri

b) Kesit üzerinde tarafsız eksenin konumunu bulunuz.

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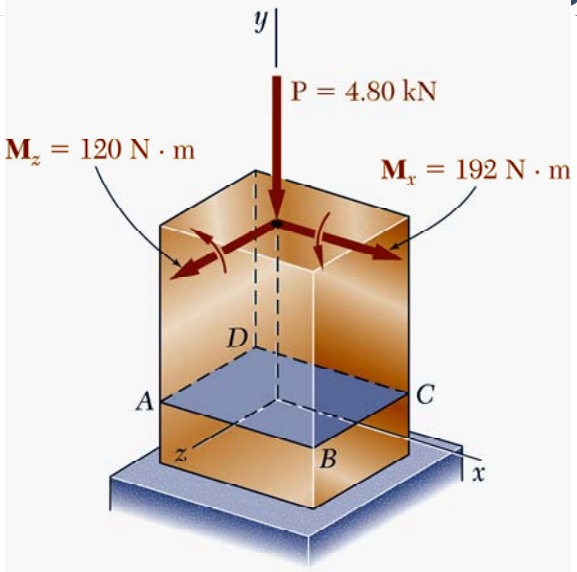

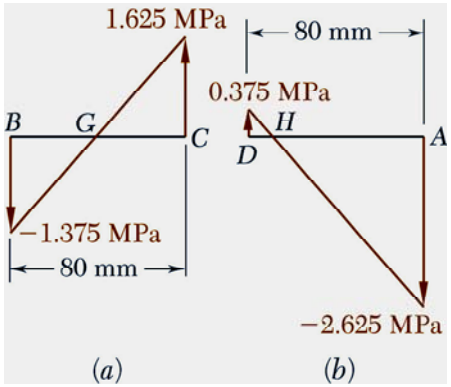

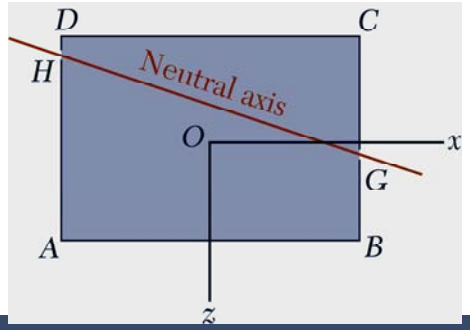

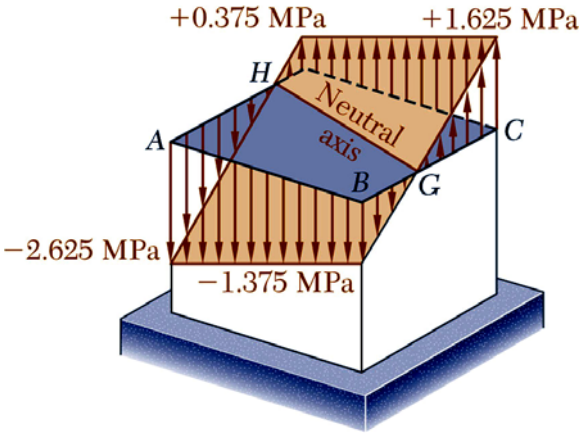

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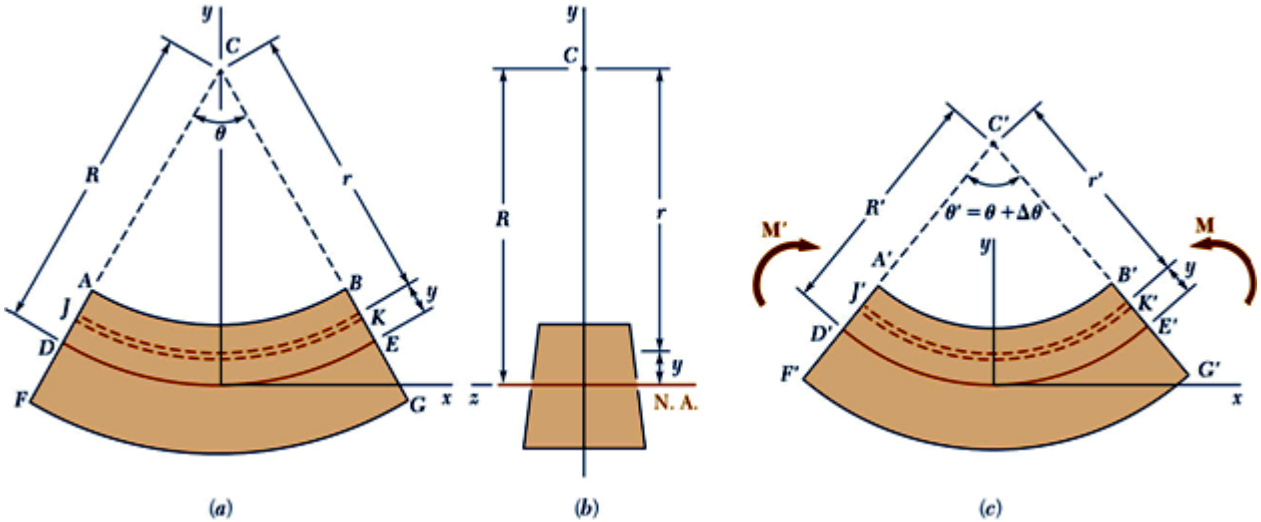
Eğri Eksenli Çubuklar (Curved Bars)



Yükleme öncesi durum

Yükleme sonrası durum

DE= tarafsız eksen/düzlem;

$$\breve{JK} > \breve{J'K'}$$

$$\breve{DE} = \breve{D'E'}$$

$$DE = R \cdot \theta$$

$$D'E' = R' \cdot \theta'$$

$$R \cdot \theta = R' \cdot \theta'$$

JK yayının boyunda meydana gelen değişim;

$$\delta = J'K' - JK = r' \cdot \theta' - r\theta$$

$$r = R - y$$

$$r' = R' - y$$

$$\delta = (R' - y)\theta' - (R - y)\theta$$

$$\delta = R' \cdot \theta' - y \cdot \theta' - R \cdot \theta + y \cdot \theta$$

$$R \cdot \theta = R' \cdot \theta' \Rightarrow \delta = -y \cdot \theta' + y \cdot \theta = -y(\theta' - \theta)$$

$$\theta' = \theta + \Delta\theta \Rightarrow \Delta\theta = \theta' - \theta$$

$$\delta = -y \cdot \Delta\theta$$

JK yayının birim şekil değiştirmesi;

$$\varepsilon_X = \frac{\delta}{L} = \frac{\delta}{JK}$$

$$\delta = -y \cdot \Delta\theta$$

$$JK = r \cdot \theta$$

$$\varepsilon_X = \frac{-y \cdot \Delta\theta}{r \cdot \theta} = \frac{-y \cdot \Delta\theta}{(R - y)\theta} = -\frac{y \cdot \Delta\theta}{(R - y)\theta}$$

$$\sigma_x = E \cdot \varepsilon_x = E\left(-\frac{y \cdot \Delta\theta}{(R - y)\theta}\right) = -E\frac{\Delta\theta}{\theta} \cdot \frac{y}{R - y}$$

$$R - r = y \Rightarrow \sigma_x = -E\frac{\Delta\theta}{\theta} \cdot \frac{R - r}{r}$$

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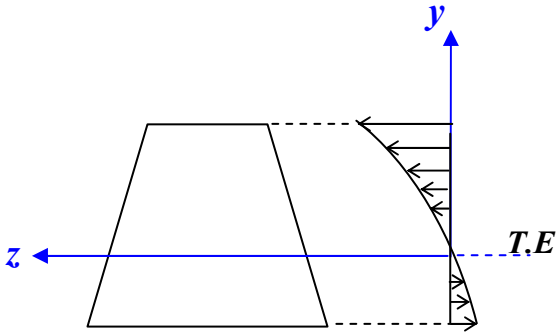
Tarafsız eksenin yeri;

$$\int \sigma_x dA = 0 \Rightarrow \int \left(-E \frac{\Delta\theta}{\theta} \cdot \frac{R-r}{r} \right) dA = 0$$

$$-E \frac{\Delta\theta}{\theta} \int \frac{R-r}{r} dA = 0 \Leftrightarrow E \neq 0, \quad \Delta\theta \neq 0, \quad \theta \neq \infty$$

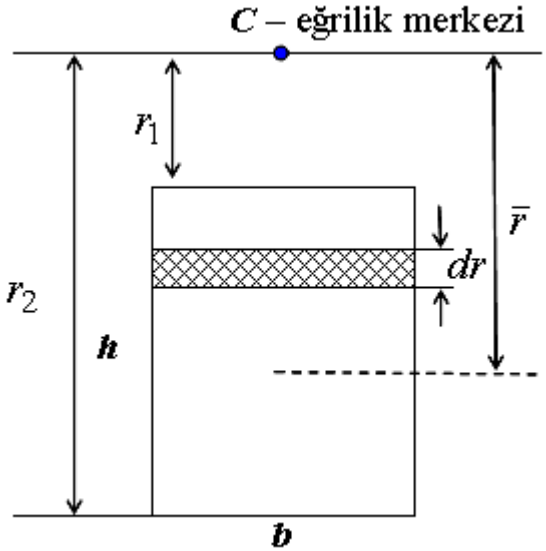
$$\int \frac{\mathrm{R-r}}{\mathrm{r}} dA = 0$$

$$\int \frac{R}{r} dA - \int dA = 0 \Rightarrow R \int \frac{dA}{r} = A \Rightarrow \boxed{R = \frac{A}{\int \frac{dA}{r}}}$$



R = Eğri eksenli bir çubuk için tarafsız eksenin yeri.

Örnek: Dikdörtgen kesite sahip eğri eksenli bir çubuk için tarafsız eksenin konumu



$$\bar{r} = \frac{r_1 + r_2}{2}$$

$$R = \frac{A}{\int \frac{dA}{r}} = \frac{bh}{\int_{r_1}^{r_2} \frac{bdr}{r}} = \frac{h}{\int_{r_1}^{r_2} \frac{dr}{r}} \Rightarrow R = \frac{h}{\ln \frac{r_2}{r_1}}$$

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Statik dengeden;

$$\int -y\sigma_x dA = M \Rightarrow \int -y\left(-\frac{E\Delta\theta}{\theta}\cdot\frac{R-r}{r}\right)dA = M$$

$$y = R - r \Rightarrow \frac{E\cdot\Delta\theta}{\theta}\int\frac{(R-r)^2}{r}dA = \frac{E\cdot\Delta\theta}{\theta}\int\frac{R^2-2Rr+r^2}{r}dA = M$$

$$\frac{E\cdot\Delta\theta}{\theta}\left[R^2\int\frac{dA}{r} - 2R\int dA + \int r dA\right] = M$$

$$\bar{r} = \frac{\int r dA}{A} \Rightarrow \int r dA = \bar{r}A$$

$$R = \frac{A}{\int\frac{dA}{r}} \Rightarrow \int\frac{dA}{r} = \frac{A}{R}$$

$$\int dA = A$$

$$\frac{E\cdot\Delta\theta}{\theta}\left[R^2\frac{A}{R} - 2RA + \bar{r}A\right] = M$$

$$\frac{E\cdot\Delta\theta}{\theta}\left(-RA + \bar{r}\cdot A\right) = M$$

$$\frac{E\cdot\Delta\theta}{\theta}A\left(\bar{r} - R\right) = M$$

$$\frac{E\cdot\Delta\theta}{\theta} = \frac{M}{A\left(\bar{r} - R\right)}$$

Elastisite modülü daima pozitiftir;

$$E > 0\,, \quad \Delta\theta > 0 \quad \text{ve} \quad \theta > 0$$

O halde eşitliğin sağ tarafı da pozitif olmalıdır:

$$\frac{M}{A(\bar{r}-R)} > 0 \Rightarrow \bar{r} - R > 0 \Rightarrow \bar{r} > R$$

$$\bar{r} - R = e$$ ***e***: eksantriklik

Bu durumda eğilmeye maruz eğri eksenli bir çubuk kesitindeki normal gerilmeler şu bağıntı ile bulunur:

$$\frac{E\cdot\Delta\theta}{\theta} = \frac{M}{A\cdot e}$$

$$\sigma_x = -\frac{E\cdot\Delta\theta}{\theta}\cdot\frac{R-r}{r} \Rightarrow \sigma_x = \frac{M}{A\cdot e}\cdot\frac{R-r}{r} = \frac{M(r-R)}{A\cdot e\cdot r}$$

$$\boxed{\sigma_x = \frac{M(r-R)}{A\cdot e\cdot r}} \quad \text{veya} \quad \sigma_x = -\frac{M\cdot y}{A\cdot e\cdot(R-y)}$$

Eğrilik Yarıçapı :

$$R\cdot\theta = R'\cdot\theta'$$

$$\theta' = \theta + \Delta\theta$$

$$\frac{1}{R'} = \frac{1}{R}\cdot\frac{\theta'}{\theta} = \frac{1}{R}\cdot\frac{\theta+\Delta\theta}{\theta} = \frac{1}{R}\left(1+\frac{\Delta\theta}{\theta}\right)$$

$$\frac{E\cdot\Delta\theta}{\theta} = \frac{M}{A\cdot e} \Rightarrow \frac{\Delta\theta}{\theta} = \frac{M}{A\cdot e\cdot E} \rightarrow \boxed{\frac{1}{R'} = \frac{1}{R}\left(1+\frac{M}{A\cdot e\cdot E}\right)}$$

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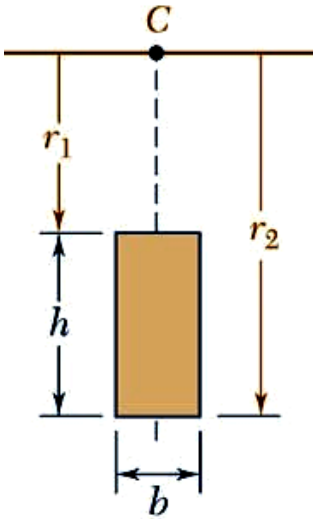
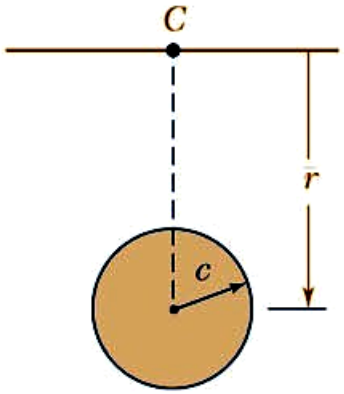


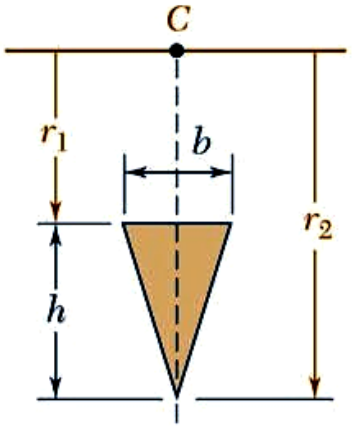
Rectangle

$$R = \frac{h}{\ln \frac{r_2}{r_1}}$$



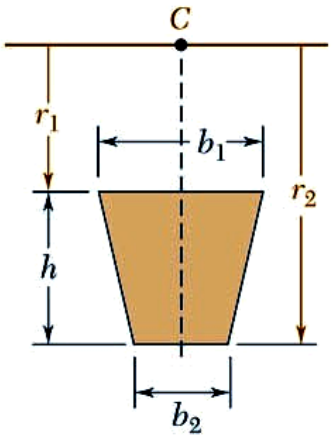
Circle

$$R = \frac{1}{2}\left(\bar{r} + \sqrt{\bar{r}^2 - c^2}\right)$$



Triangle

$$R = \frac{\frac{1}{2}h}{\frac{r_2}{h}\ln\frac{r_2}{r_1} - 1}$$



Trapizoid

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

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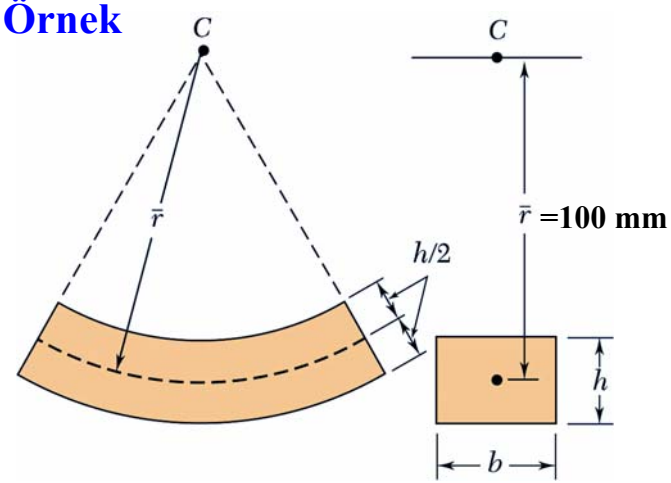
Örnek



b=50 mm, h=25 mm olduğuna göre; şekildeki eğri eksenli çubuğa M=500 N.m'lik bir moment uygulandığında;

a) Kesitin geometrik merkezi ile tarafsız eksen arasındaki mesafeyi (***e***) bulunuz.

b) Meydana gelen en büyük çeki ve bası gerilmelerini bulunuz.

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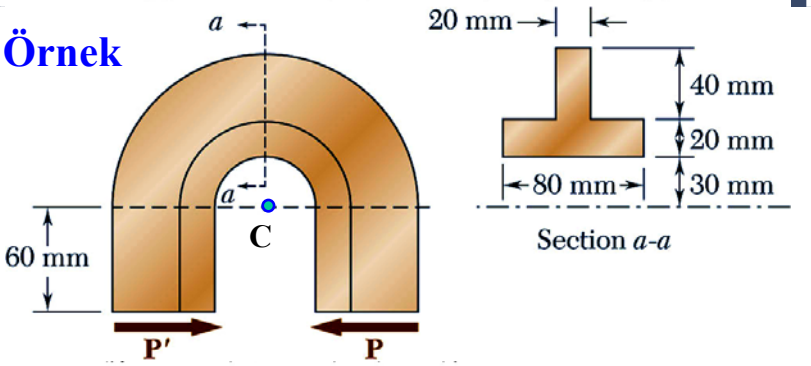
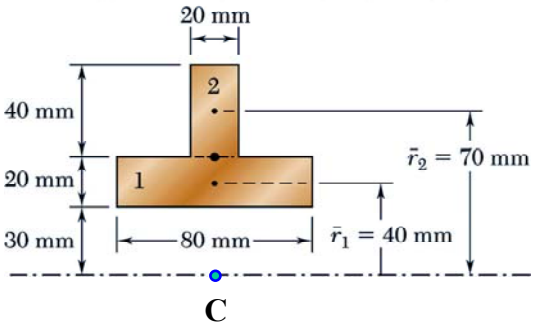
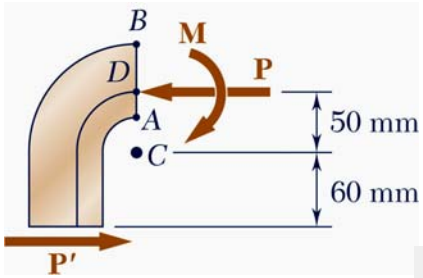
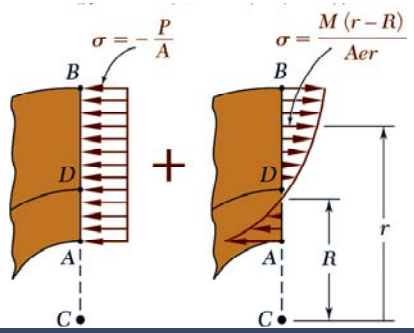
Örnek

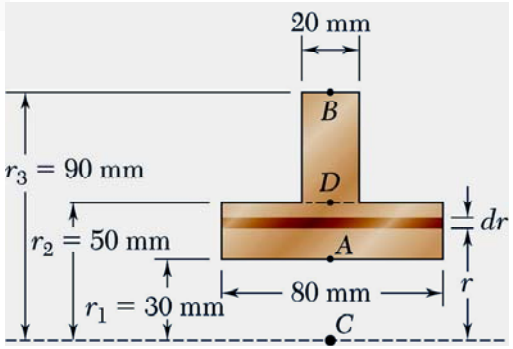


Şekildeki eğri eksenli ve T kesitli çubuğun emniyetli bası gerilmesi değeri 50 MPa olduğuna göre uygulanabilecek en büyük **P** kuvvetinin değerini bulunuz.









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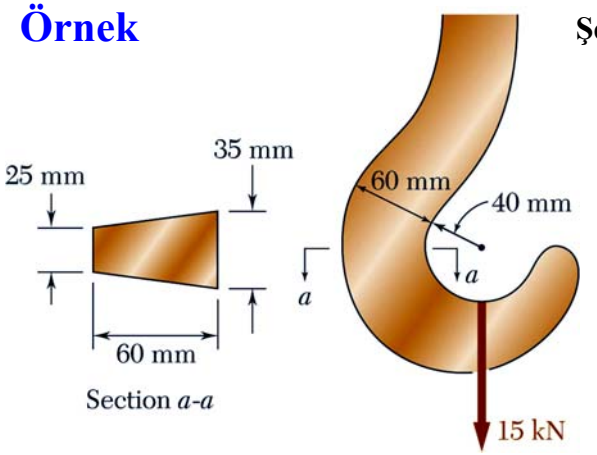
Örnek

Şekildeki taşıyıcı kancanın *a-a* kesitinde meydana gelen en büyük çeki gerilmesini bulunuz.



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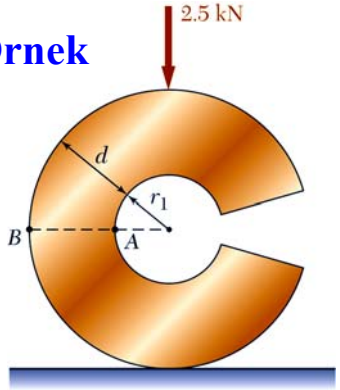
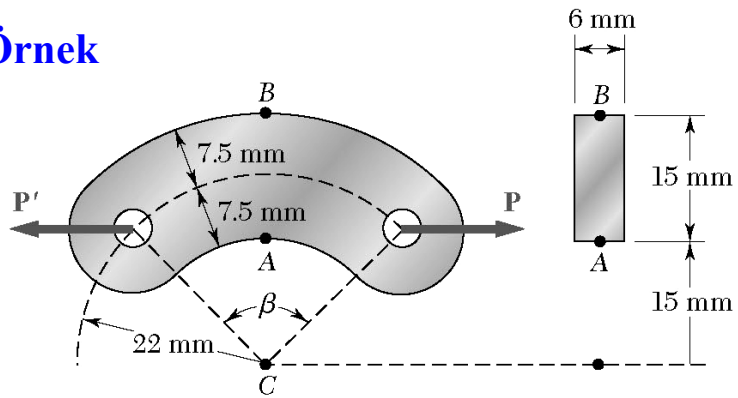
Uygulama için örnek problemler

Örnek



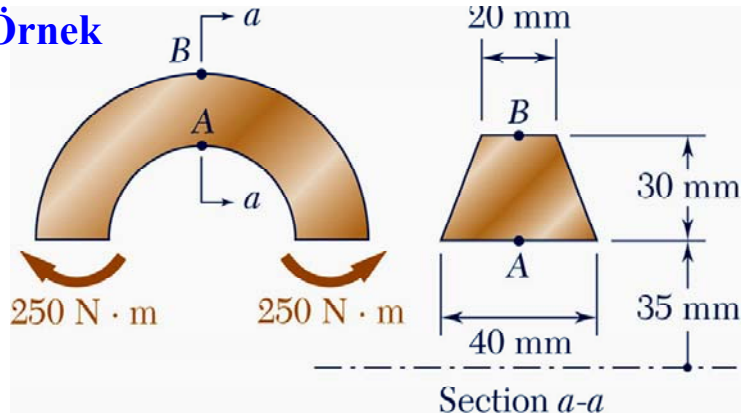
Kesit çapı d=32 mm olan şekildeki eğri eksenli çubuğun ***A*** ve ***B*** noktalarında meydana gelen gerilmeleri bulunuz, r_1=20 mm.

Örnek



Şekildeki eğri eksenli ve dikdörtgen kesitli elamanın emniyetli gerilme değeri 100 MPa olduğuna göre uygulanabilecek en büyük **P** kuvvetinin değerini bulunuz.

Örnek



Şekildeki eğri eksenli çubuğun ***A*** ve ***B*** noktalarında meydana gelen gerilmeleri bulunuz

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