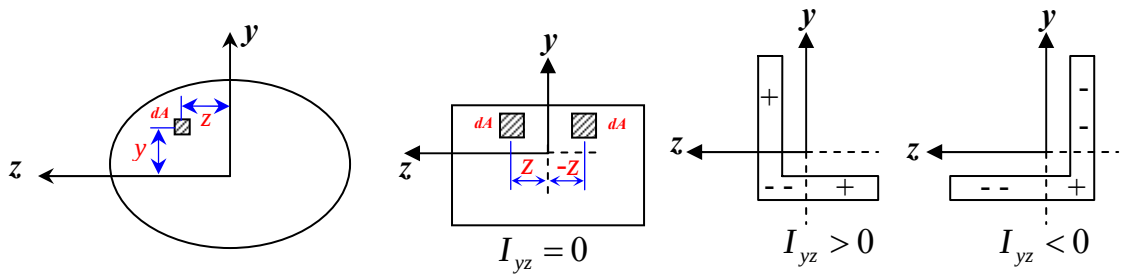
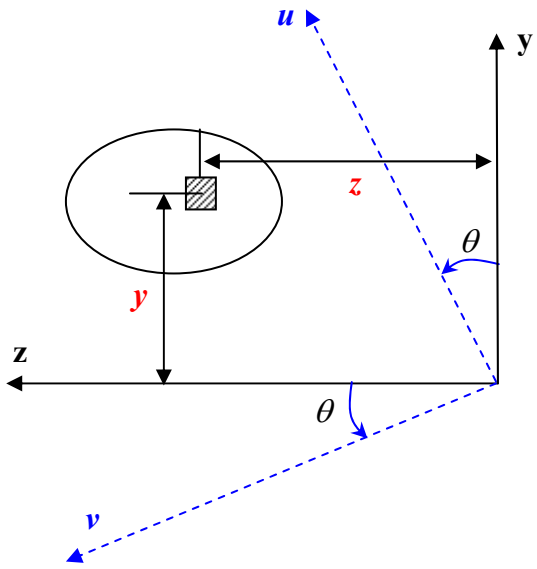
ASAL ATALET MOMENTLERİ



$$I_y = \int z^2 dA$$
$$I_z = \int y^2 dA$$
$$I_{yz} = \int yz dA$$

$I_{yz} = 0$ olduğunda; yani ***y*** veya ***z*** eksenlerinden en az bir tanesi kesitin simetri ekseni ise, y ve ***z*** asal atalet eksenlerini ve $\boldsymbol{I_y}$ ve $\boldsymbol{I_z}$ bu eksenlere göre asal atalet momentlerini ($\boldsymbol{I_{max}}$, $\boldsymbol{I_{min}}$) ifade eder.

Genel olarak ***y*** ve ***z*** eksenlerine göre atalet momentleri biliniyorsa, kesitin herhangi bu eksenlerle $\boldsymbol{\theta}$ açısı yapan başka eksenlerle göre atalet momentleri ve asal atalet (max, min) momentleri şöyle bulunur:



$$u = y\cos\theta + z\sin\theta$$
$$v = z\cos\theta - y\sin\theta$$
$$I_u = \int v^2 dA = \int (z\cos\theta - y\sin\theta)^2 dA$$
$$= \cos^2\theta \int z^2 dA + \sin^2\theta \int y^2 dA - 2\sin\theta\cos\theta \int yz dA$$
$$I_u = I_y \cos^2\theta + I_z \sin^2\theta - 2I_{yz}\sin\theta\cos\theta$$

$$I_v = I_z \cos^2\theta + I_y \sin^2\theta + 2I_{yz}\sin\theta\cos\theta$$
$$I_{uv} = \int uv dA = \int (y\cos\theta + z\sin\theta)(z\cos\theta + y\sin\theta) dA$$
$$I_{uv} = -(I_z - I_y)\sin\theta\cos\theta + I_{yz}(\cos^2\theta - \sin^2\theta)$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2} \qquad \sin^2\theta = \frac{1-\cos 2\theta}{2} \qquad \sin 2\theta = 2\sin\theta\cos\theta$$

$$I_v = \frac{I_z + I_y}{2} + \frac{I_z - I_y}{2}\cos 2\theta + I_{yz}\sin 2\theta$$

$$I_u = \frac{I_z + I_y}{2} - \frac{I_z - I_y}{2}\cos 2\theta - I_{yz}\sin 2\theta$$

$$I_{uv} = -\frac{I_z - I_y}{2}\sin 2\theta + I_{yz}\cos 2\theta$$

$$\boxed{I_u + I_v = I_z + I_y}$$

Asal Atalet Momentleri ve Eksen doğrultuları (maksimum-minimum problemi !**):**

$$\frac{dI_v}{d\theta} = \frac{I_z - I_y}{2}(-\sin 2\theta)\cdot 2 + I_{yz}(\cos 2\theta)\cdot 2 = 0$$

$$\boxed{tg2\theta_m = \frac{I_{yz}}{\frac{I_z - I_y}{2}} \quad , \quad I_{\max,\min} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_z - I_y}{2}\right)^2 + I^2{}_{yz}}}$$

$$I_z > I_y \qquad I_{\max} = I_v \quad , \; I_{\min} = I_u$$

$$I_y > I_z \qquad I_{\max} = I_u \quad , \; I_{\min} = I_v$$

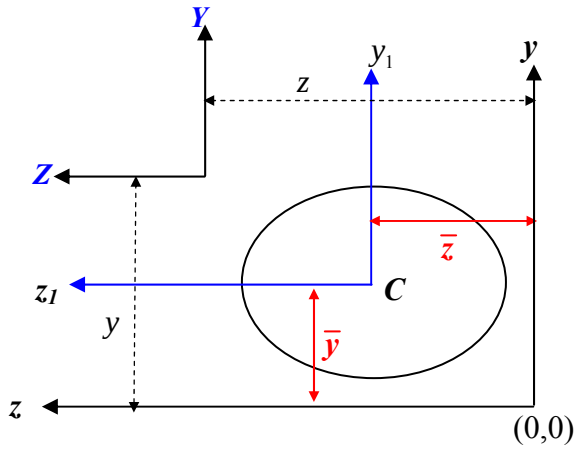
Paralel Eksenler Teoremi

Bir kesitin geometrik merkezinden geçen ***y_1 ve z_1*** eksenlerine göre atalet momentleri ***I_{y1}*** ve ***I_{z1}*** ise bu eksenlere paralel ***Y*** ve ***Z*** eksenlerine göre atalet momentleri (***I_Y*** ve ***I_Z***) paralel eksenler teoremi ile şu şekilde bulunur:

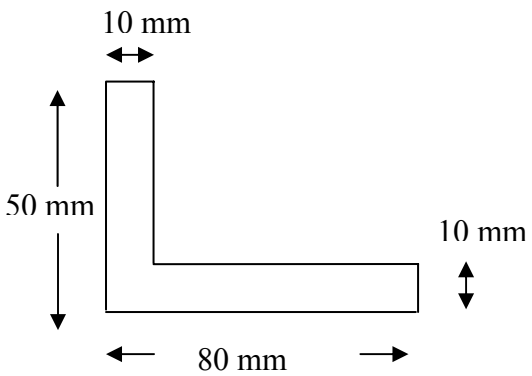
$$I_Y = I_{y1} + A(\bar{y} - y)^2$$

$$I_Z = I_{z1} + A\,(\bar{z} - z)^2$$

$$I_{YZ} = I_{y1z1} + A(\bar{y} - y)(\bar{z} - z)$$



Örnek Problem:



a) Şekilde verilen kesitin geometrik merkezini

b) Geometrik merkezinden geçen eksenlere göre atalet momentlerini

c) Asal atalet momentlerini bulunuz kesit üzerinde gösteriniz.

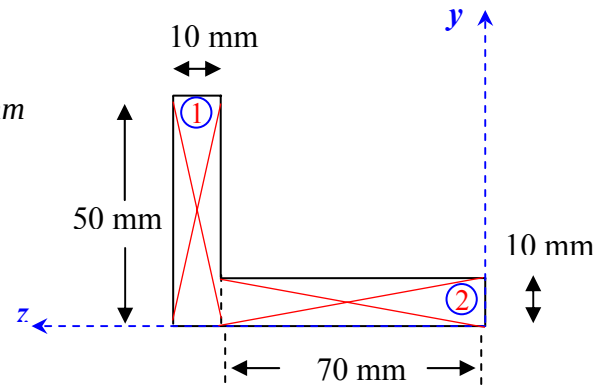
$$\bar{z} = \frac{A_1 z_1 + A_2 z_2}{A_1 + A_2} = \frac{50 \cdot 10 \cdot 75 + 70 \cdot 10 \cdot 35}{500 + 700} = 51.67mm$$

$$\bar{y} = 13.33mm$$

$$I_{Z1} = \frac{10 \cdot 50^3}{12} + 500(13{,}33 - 25)^2 = 172261mm^4$$

$$I_{Y1} = \frac{50 \cdot 10^3}{12} + 500(51.67 - 75)^2 = 276311mm^4$$

$$I_{Y1Z1} = 0 + (50 \cdot 10)(13.37 - 25)(51.67 - 75) = 136130mm^4$$



$$I_{Z2} = 54405mm^4$$

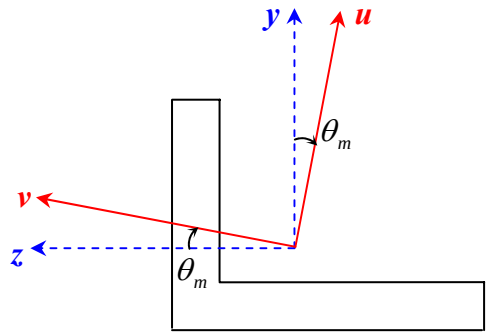
$$I_{Y2} = 480355mm^4$$

$$I_{Y2Z2} = 97203mm^4$$

$$I_Z = I_{Z1} + I_{Z2} = 226666mm^4$$

$$I_Y = I_{Y1} + I_{Y2} = 756666mm^4$$

$$I_{YZ} = 233333mm^4$$



$$tg2\theta_m = \frac{I_{YZ}}{\frac{I_Z - I_Y}{2}} = \frac{233333}{\frac{226666 - 756666}{2}} \Rightarrow \theta_m = -20.68^o$$

$$I_{\max,\min} = I_{u,v} = \frac{226666 + 756666}{2} \pm \sqrt{\left(\frac{226666 - 756666}{2}\right)^2 + (233333)^2}$$

$$I_{\max} = I_u = 844751mm^4$$

$$I_{\min} = I_v = 138581mm^4$$