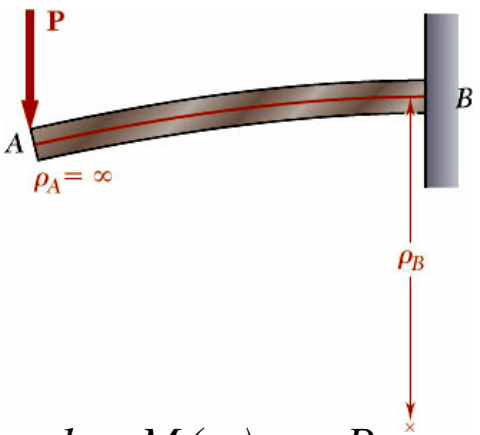
KİRİŞLERDE ÇÖKME



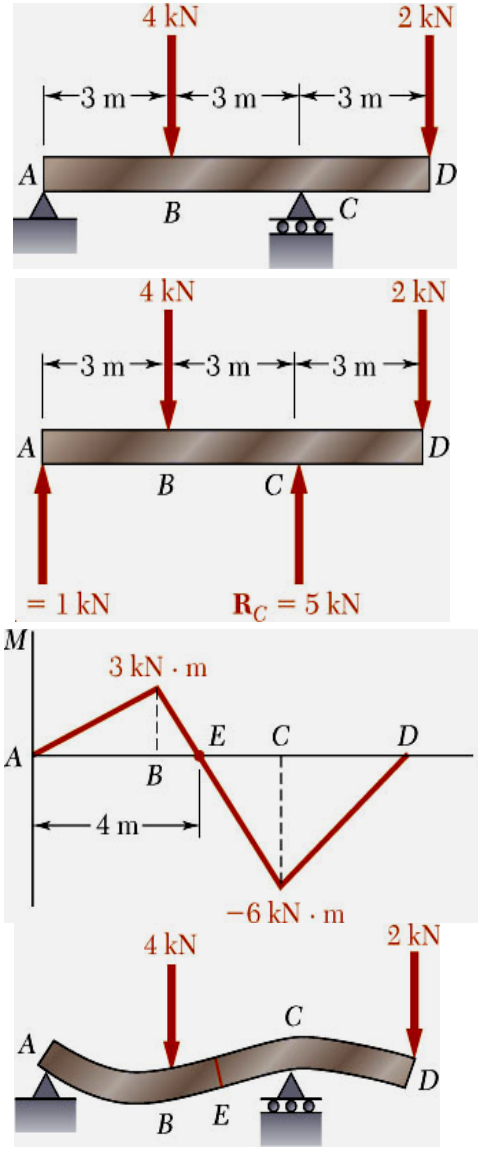
$$\frac{1}{\rho} = \frac{M(x)}{EI} = -\frac{Px}{EI}$$

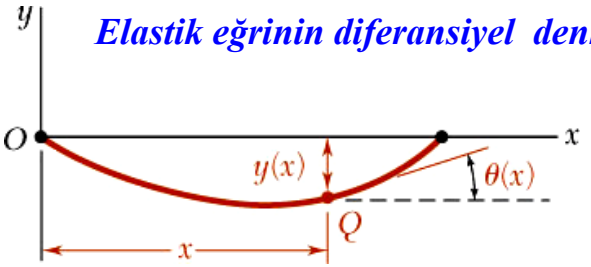
- Serbest uçta (*A*),

$$\frac{1}{\rho_A} = 0, \qquad \rho_A = \infty$$

- Ankastre uçta (*B*),

$$\frac{1}{\rho_B} \neq 0, \quad |\rho_B| = \frac{EI}{PL}$$





Elastik eğrinin diferansiyel denklemi

Küçük çökmeler için;

$$\frac{1}{\rho} = \frac{\dfrac{d^2y}{dx^2}}{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{3/2}} \approx \frac{d^2y}{dx^2}$$

$$\frac{1}{\rho} = \frac{d^2y}{dx^2} = \frac{M(x)}{EI} \rightarrow \boxed{EI\frac{d^2y}{dx^2} = M(x)}$$

- İntegral alınarak eğim (***θ=dy/dx***) ve çökmenin (***y***) denklemleri bulunur. Sınır şartları yazılarak integral sabitleri bulunur.

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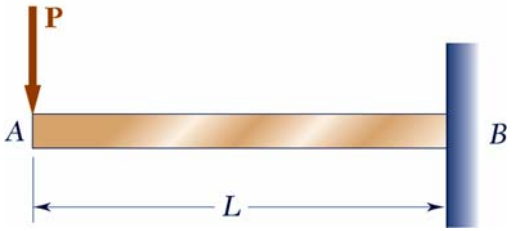
Örnek

Integrasyon Metodu ile Çözümler



Şekildeki yükleme durumu için kirişin A noktasındaki çökmeyi ve eğim açısını bulunuz.

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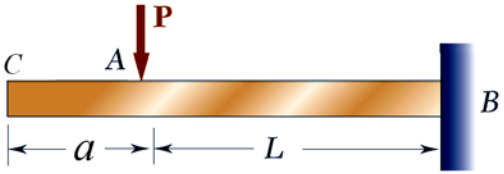
Örnek: Şekildeki yükleme durumu için kirişin C noktasındaki çökmeyi ve eğim açısını bulunuz.



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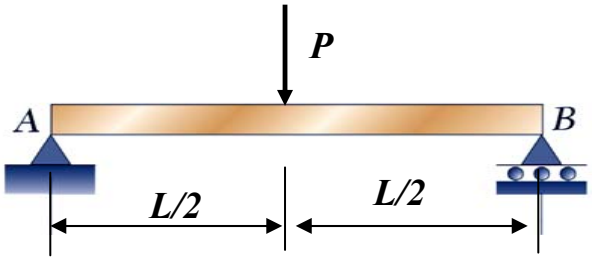
Örnek: Şekildeki yükleme durumu için maksimum çökmeyi ve eğim açısını bulunuz.



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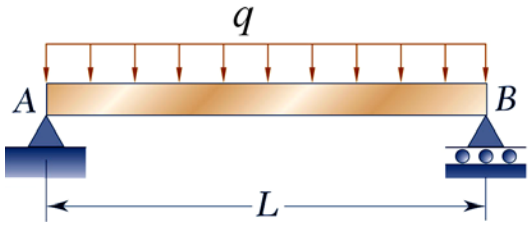
Örnek: Şekildeki yükleme durumu için kirişteki maksimum çökmeyi A noktasındaki eğim açısını bulunuz.



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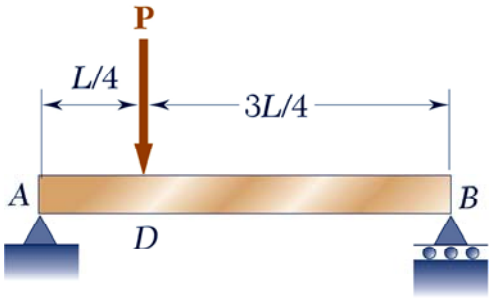
Örnek: Şekildeki yükleme durumu için kirişin D noktasındaki çökmeyi ve eğim açısını bulunuz.



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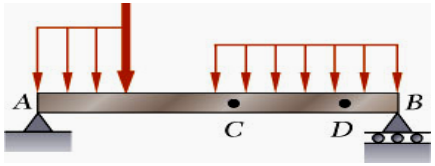
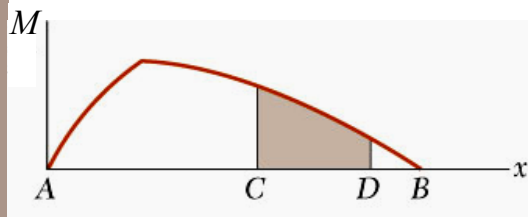
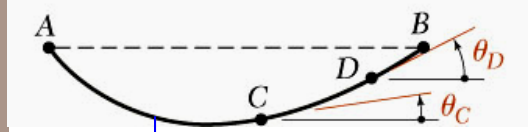
Moment-Alanı Metodu



Değişik yüklere maruz bir kiriş göz önüne alınırsa:



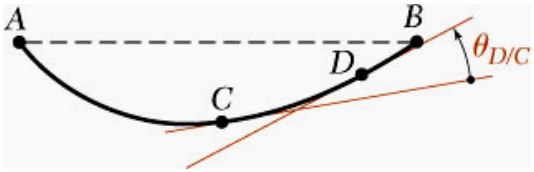
$$\frac{d\theta}{dx} = \frac{d^2y}{dx^2} = \frac{M}{EI}$$



$$\int_{\theta_C}^{\theta_D} d\theta = \int_{x_C}^{x_D} \frac{M}{EI}\,dx$$

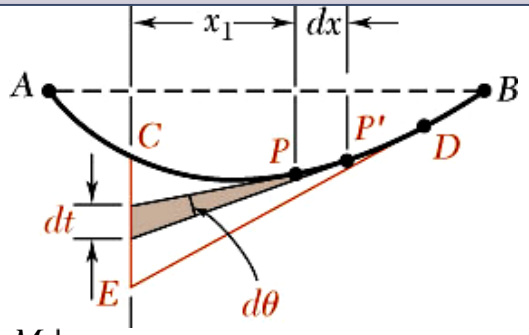
Çökme eğrisi

$$\theta_D - \theta_C = \int_{x_C}^{x_D} \frac{M}{EI}\,dx$$

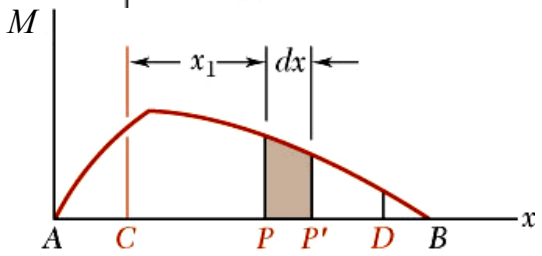


***1. Moment-Alanı Teoremi*:**

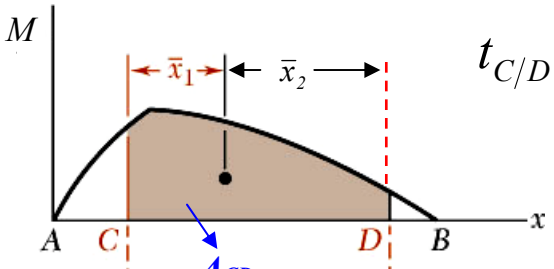
$$\theta_{D/C} = \frac{\textbf{\textit{C}} \text{ ve } \textbf{\textit{D}} \text{ arasındaki moment alanı}}{\textbf{\textit{EI}}}$$



$$dt = x_1 d\theta = x_1 \frac{M}{EI}\,dx$$



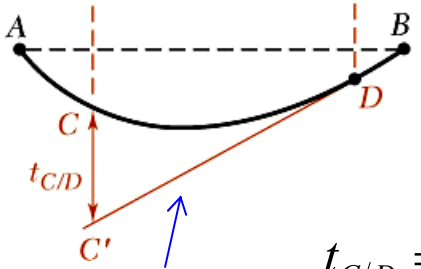
$$t_{C/D} = \int_{x_C}^{x_D} x_1 \frac{M}{EI}\,dx$$



$t_{C/D}$ = C noktasının D'ye göre teğetsel değişimi

$$t_{C/D} = \frac{A_{CD} \cdot \bar{x}_1}{EI}$$

$$t_{D/C} = \frac{A_{CD} \cdot \bar{x}_2}{EI}$$



Referans teğet

***2. Moment-Alanı Teoremi*:**

$$t_{C/D} = \frac{\textbf{\textit{C}} \text{ ve } \textbf{\textit{D}} \text{ arasındaki moment alanının C'ye göre statik momenti}}{\textbf{\textit{EI}}}$$

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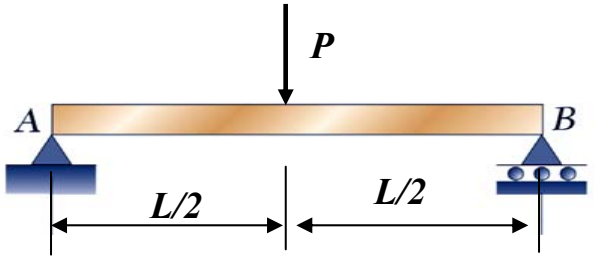
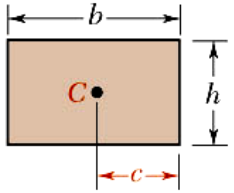
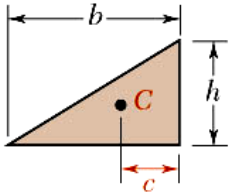
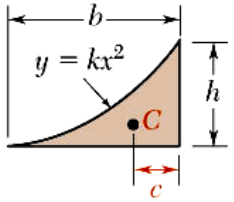
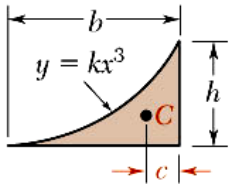
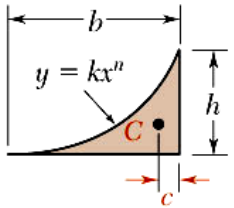
Örnek:



Simetrik Yükleme:

Shape		Area	c
Rectangle		bh	$\frac{b}{2}$
Triangle		$\frac{bh}{2}$	$\frac{b}{3}$
Parabolic spandrel		$\frac{bh}{3}$	$\frac{b}{4}$
Cubic spandrel		$\frac{bh}{4}$	$\frac{b}{5}$
General spandrel		$\frac{bh}{n+1}$	$\frac{b}{n+2}$

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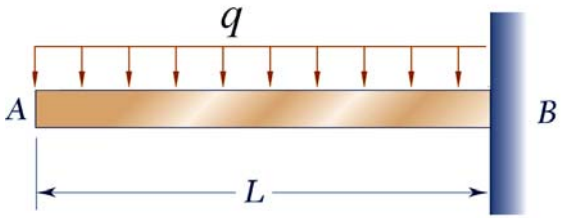
Örnek:



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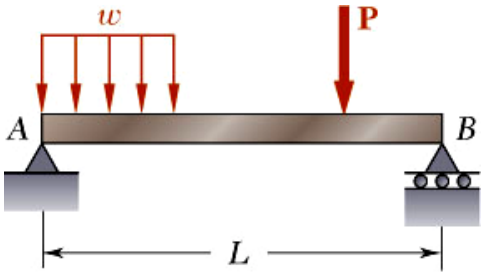
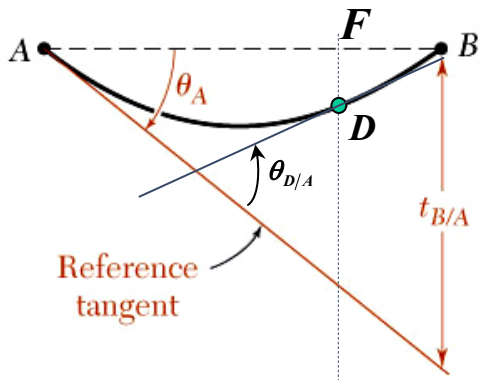
Simetrik olmayan Yükleme:



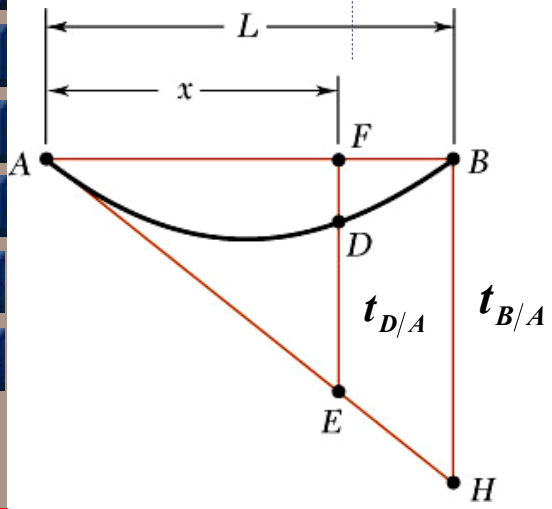


$$\tan\theta_A \approx \theta_A = -\frac{t_{B/A}}{L}$$

Saat ibreleri yönündeki açılar (-), zıt yön (+) alınır.

$$\theta_D = \theta_A + \theta_{D/A}$$

$$y_D = FD = ?$$

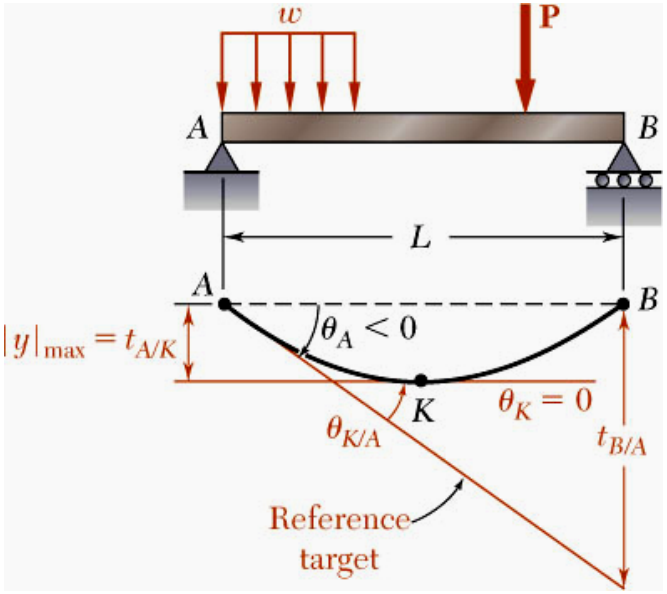


$$\frac{EF}{x} = \frac{HB}{L}; \quad EF = \frac{x}{L}t_{B/A}$$

$$y_D = ED - EF$$

$$y_D = t_{D/A} - \frac{x}{L}t_{B/A}$$

Maksimum Çökme



Maksimum çökmenin olduğu bir K noktasında eğim "0" dır. Yani, teğet yataydır.

$$\theta_A = -\frac{t_{B/A}}{L}$$

$$\theta_K = 0 = \theta_A + \theta_{K/A} \qquad \theta_{K/A} = -\theta_A$$

y_{max} değeri, A ve K arasındaki moment alanının A dan geçen düzey eksene göre statik momentinin EI değerine bölünmesi ile bulunabilir.

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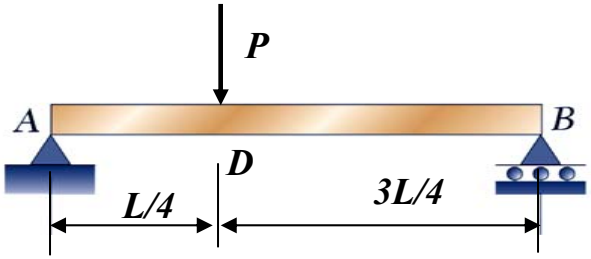
Örnek: Şekildeki yükleme durumu için kirişin D noktasındaki çökmeyi, eğim açısını ve maksimum çökmeyi bulunuz.



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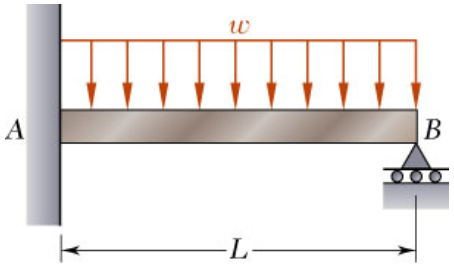
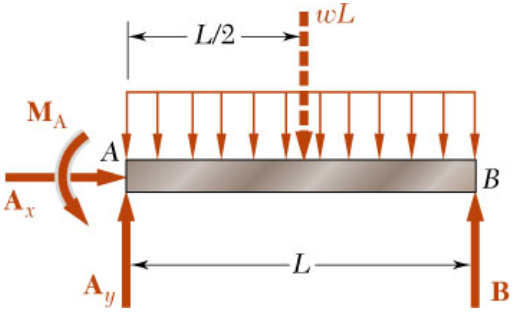
Statikçe belirsiz (hiperstatik) sistemler

Örnek: Şekildeki yükleme durumu için mesnet noktalarındaki reaksiyon kuvvetlerini bulunuz.





Statik Denge:

4 bilinmeyen: A_x, A_y, B, M_A

3 denklem: $\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_A = 0$

Çözüm için sınır şartlarından yararlanarak ilave denklem/denklemler yazmak gerekir.

End

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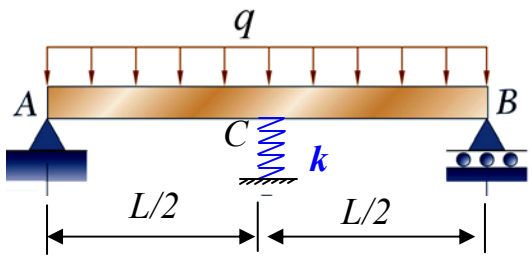
Örnek: Şekildeki yükleme durumu için yay kuvvetini bulunuz.



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