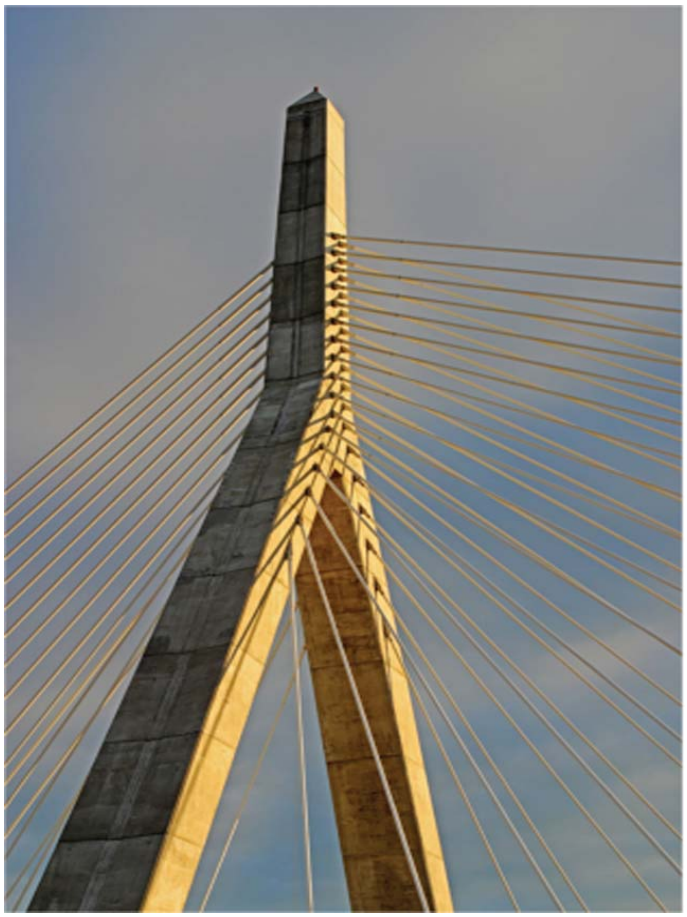
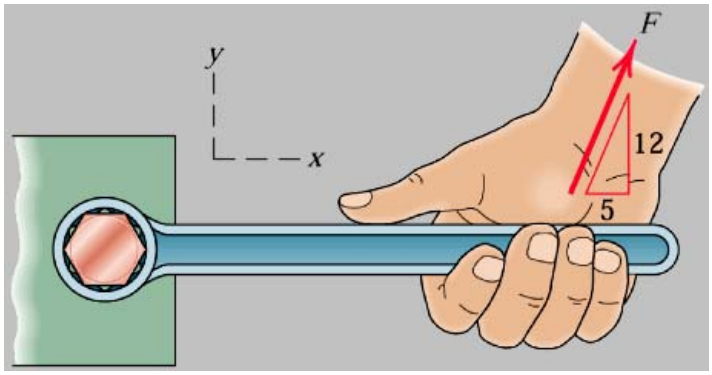
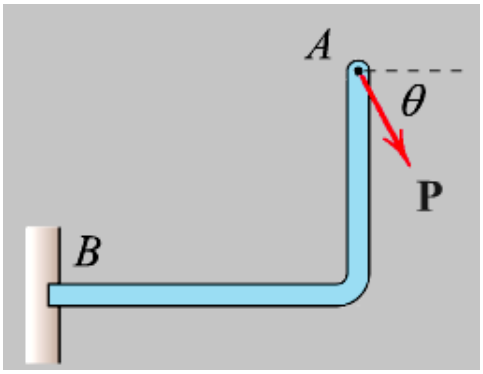
2. KUVVET SİSTEMLERİ

2.1 Giriş

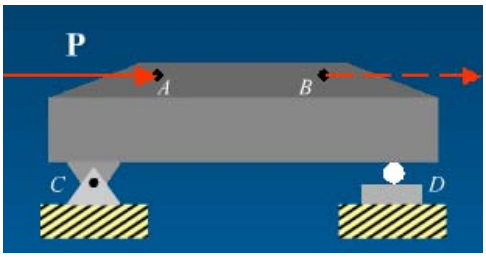




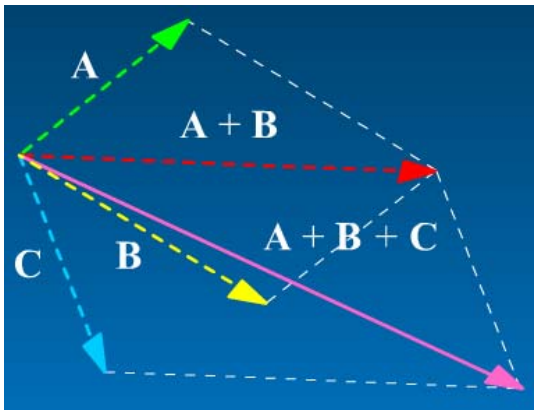


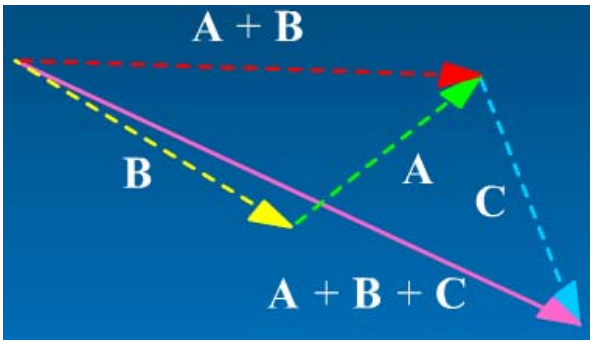
Kuvvet: Şiddet (P), doğrultu (θ) ve uygulama noktası (A) ile karakterize edilen ve bir cismin diğerine uyguladığı "itme" veya "çekme" olarak tanımlanabilir. Bu parametrelerden birinin değişimi B notasındaki mesnet tepkilerini değiştirecektir.

Taşınabilirlik Prensibi: Bir kuvvet etki çizgisi üzerindeki herhangi bir noktaya taşınabilir (kayan vektör). Bundan dış reaksiyonlar (mesnet tepkileri)etkilenmez, ancak iç etkiler değişir (mukavemet dersinde incelenecek).



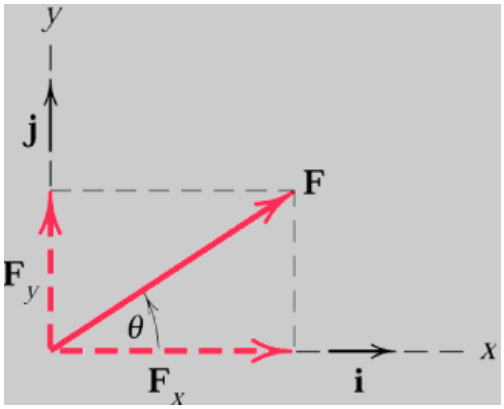
Paralelkenar kuralı v e Üçgen kuralı:





2.2 İki Boyutlu (2-D) Kuvvet Sistemleri

2.2.1 Dik Bileşenler:



i, **j**: birim vektörler

$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$$
$$\mathbf{F} = F_x\,\mathbf{i} + F_y\,\mathbf{j}$$

Burada; $\mathbf{F_x}$ ve $\mathbf{F_y}$ kuvvetin vektör bileşenlerini, F_x *ve* F_y ise bileşenlerin şiddetini (skaler bileşen)temsil etmektedir.

Skaler bileşenler:

$$F_x = F\cos\theta$$
$$F_y = F\sin\theta$$

$$F^2 = F_x^2 + F_y^2$$

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

Vektör toplamında skaler bileşenlerin toplanması:

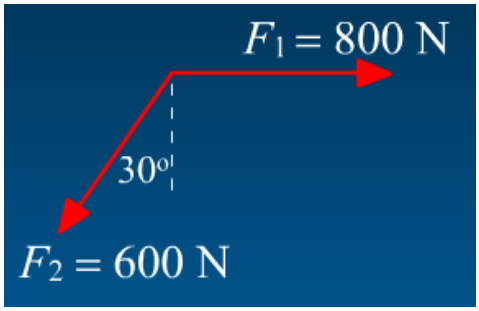
$$\mathbf{R} = R_x\,\mathbf{i} + R_y\,\mathbf{j}$$

$$\Sigma\mathbf{F} = \sum_{i=1}^{n}(F_{i,x}\,\mathbf{i} + F_{i,y}\,\mathbf{j}) = (\Sigma F_x)\,\mathbf{i} + (\Sigma F_y)\,\mathbf{j}$$

$$R_x = \Sigma F_x$$

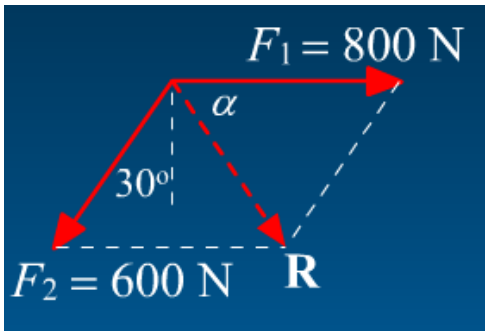
$$R_y = \Sigma F_y$$

Örnek Problem:



Şekilde verilen iki vektörü
a) Paralelkenar yöntemi
b) Dik bileşenler yöntemini
Kullanarak toplayınız.

a)Paralelkenar yöntemi ile:



Kosinüs Teoremi

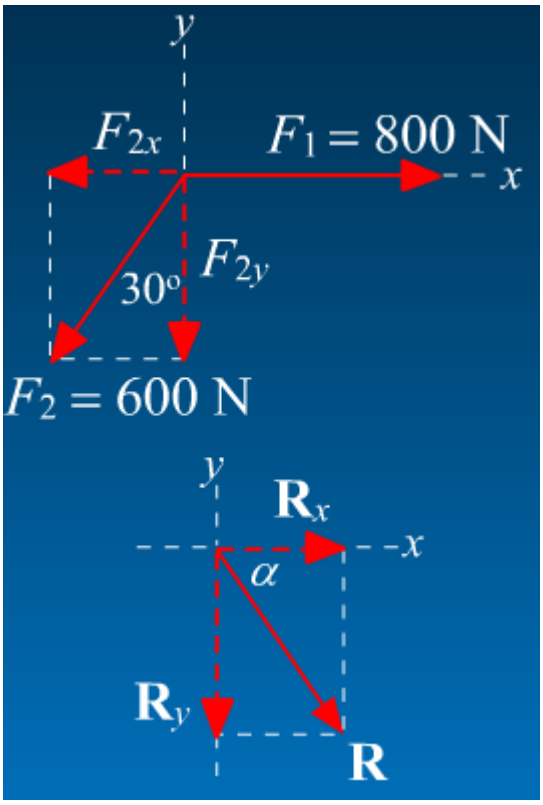
$$R^2 = 800^2 + 600^2 - 2\,(800)\,(600)\cos 60^\circ$$

$$R = 721 \text{ N}$$

Sinüs Teoremi:

$$\frac{\sin\alpha}{600} = \frac{\sin 60^\circ}{721}\ ,\ \alpha = 46.1^\circ$$

b) Dik bileşenleri toplayarak:



$$R_x = \Sigma F_x = 800 - 600\sin 30^\circ = 500 \text{ N}$$

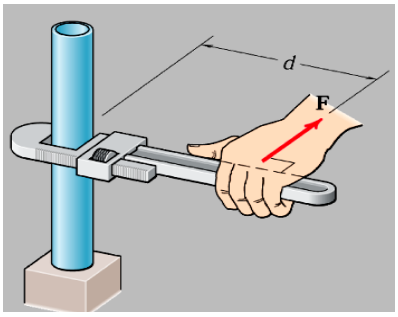
$$R_y = \Sigma F_y = 0 - 600\cos 30^\circ = -520 \text{ N}$$

$$\mathbf{R} = R_x\,\mathbf{i} + R_y\,\mathbf{j} = (500\,\mathbf{i} - 520\,\mathbf{j}) \text{ N}$$

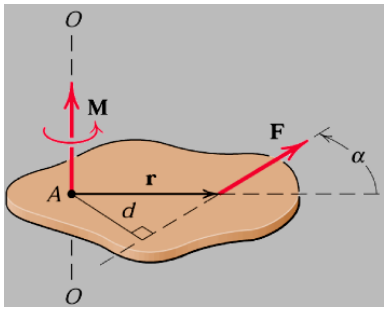
$$R = \sqrt{R_x^2 + R_y^2} = 721 \text{ N}$$

$$\alpha = \tan^{-1}\frac{|R_y|}{|R_x|} = 46.1^\circ$$

2.2.2 Moment:



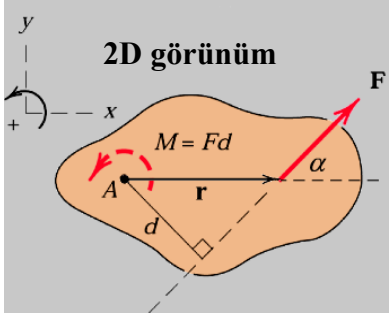
Bir kuvvetin bir cismi herhangi bir eksen etrafında döndürme etkisine (eğilimine) moment denir. Bu etki **F**'in şiddetine ve dönme eksenine olan dik mesafeye (moment kolu) "*d*" bağlıdır.



F'in A noktası etrafındaki momenti (**M**$_A$ moment vektörü);

$$\mathbf{M}_A = \mathbf{r} \times \mathbf{F}$$

Moment yönü sağ el kuralı ile bulunur.



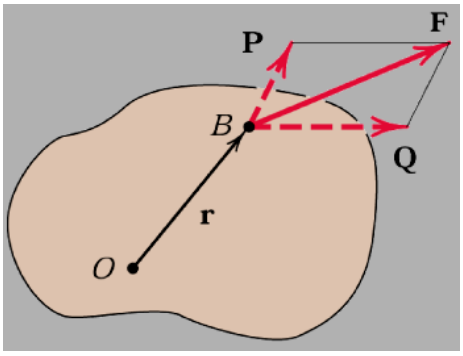
Burada "**r**" A noktasından başlayıp **F**'in etki çizgisi üzerindeki herhangi bir noktada biten bir konum vektörüdür.

M$_A$ momentinin şiddeti (M_A),

$$M_A = rF\sin\alpha = F(r\sin\alpha) = Fd$$

şeklinde bulunur.

Varignon Teoremi:

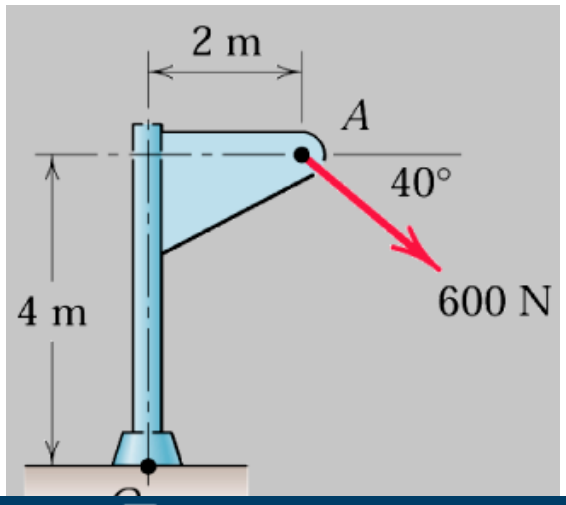


$$\begin{aligned}\mathbf{M}_O &= \mathbf{r} \times \mathbf{F} \\ &= \mathbf{r} \times (\mathbf{P} + \mathbf{Q}) \\ &= \mathbf{r} \times \mathbf{P} + \mathbf{r} \times \mathbf{Q}\end{aligned}$$

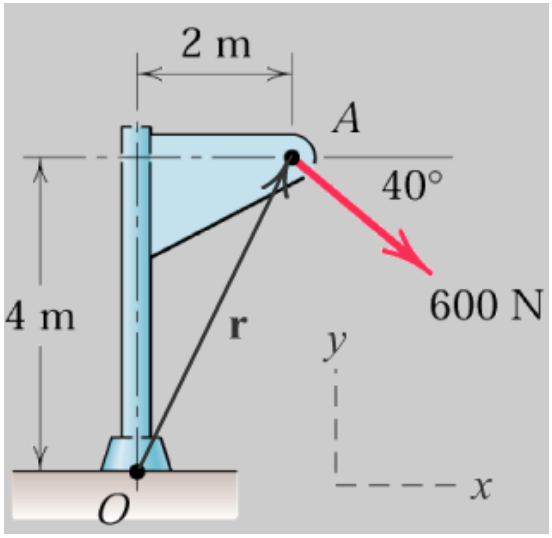
Yani bir vektörün bir noktaya göre momenti, bileşenlerin bu noktaya göre momentlerinin toplamına eşittir.

Örnek Problem: Şekildeki 600 N'luk kuvvetin O noktasına göre momentini 5 farklı yolla bulalım.

Çözüm: Bu problemi beş farklı yol ile çözebiliriz.



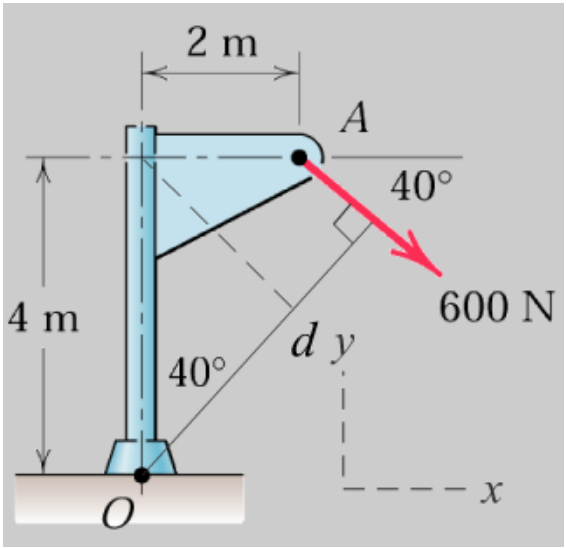
I. Yol



$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$
$$= (2\,\mathbf{i} + 4\,\mathbf{j})$$
$$\times\ 600\,(\cos 40^\circ\,\mathbf{i} - \sin 40^\circ\,\mathbf{j})$$
$$= -2610\,\mathbf{k}\ \mathrm{N \cdot m}$$
$$M_O = 2610\ \mathrm{N \cdot m}$$

II. Yol

$$M = Fd$$
$$d = 4\cos 40^\circ + 2\sin 40^\circ$$
$$= 4.35\ \mathrm{m}$$
$$\circlearrowleft + \ M = 600\,(4.35) = 2610\ \mathrm{N \cdot m}$$



III. Yol

Varignon Teoremi

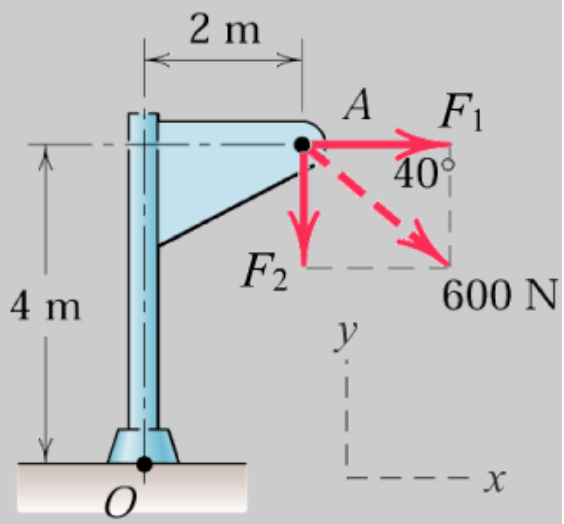
$F_1 = 600 \cos 40^o = 460$ N

$F_2 = 600 \sin 40^o = 386$ N

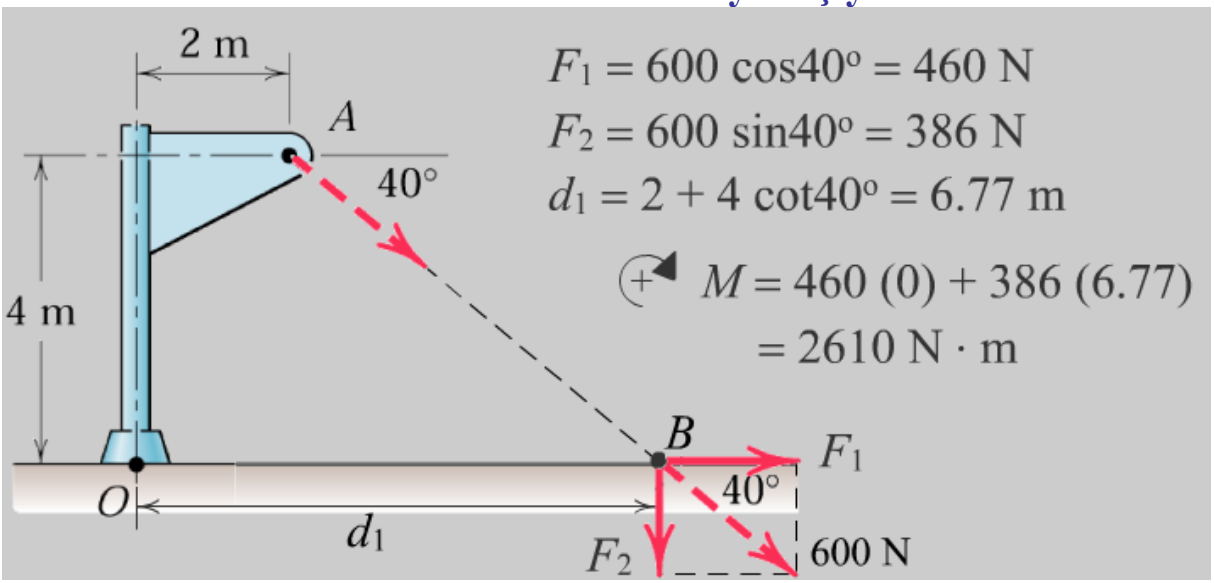
Then

$\circlearrowleft + \; M = 460\,(4) + 386\,(2)$

$= 2610$ N · m



IV. Yol : Kuvveti zemindeki bir noktaya taşıyarak



$F_1 = 600 \cos 40^o = 460$ N

$F_2 = 600 \sin 40^o = 386$ N

$d_1 = 2 + 4 \cot 40^o = 6.77$ m

$\circlearrowleft + \; M = 460\,(0) + 386\,(6.77)$

$= 2610$ N · m

V. Yol : Kuvveti O'nun üzerinde bir noktaya taşıyarak:

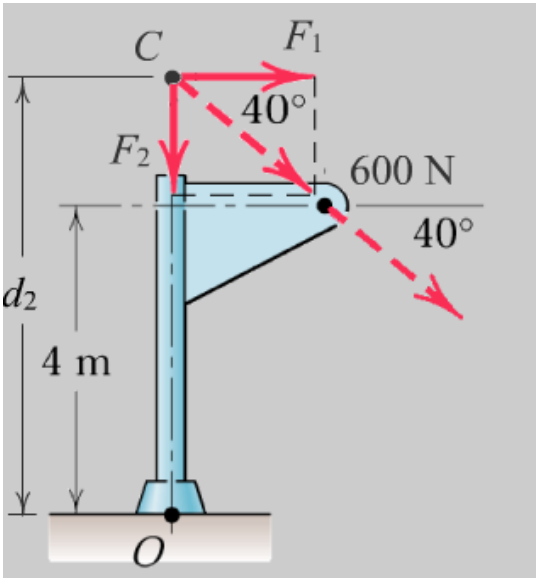
$F_1 = 600 \cos 40^o = 460$ N

$F_2 = 600 \sin 40^o = 386$ N

$d_2 = 4 + 2 \tan 40^o = 5.68$ m

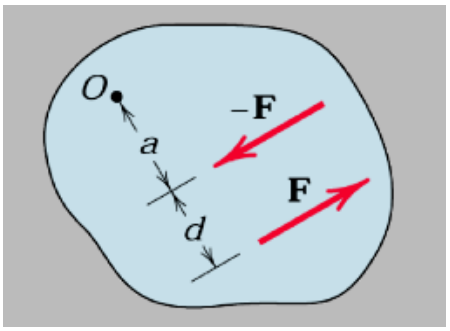
$\circlearrowleft + \; M = 460\,(5.68) + 386\,(0)$

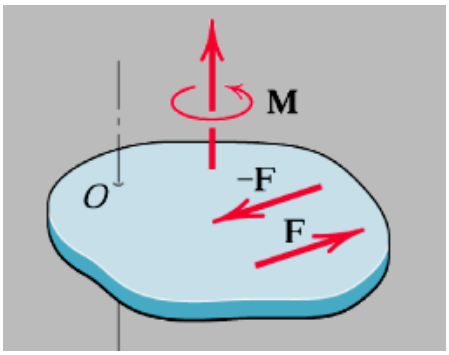
$= 2610$ N · m



2.2.3 Kuvvet Çifti (Couple)

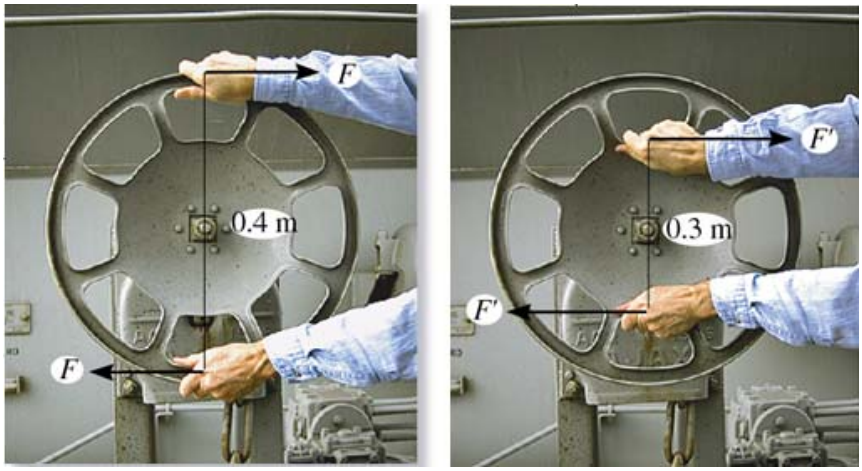
Bir **kuvvet çifti** şiddetleri eşit fakat yönleri zıt iki paralel kuvvetten meydana gelir.





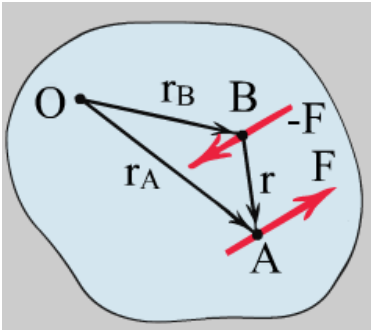
$$M_O = F(a + d) - Fa = Fd$$

Yani, kuvvet çiftinin momenti moment merkezinden (referans O noktası) bağımsızdır. Bu nedenle kuvvet çiftinin momenti bir "Serbest Vektör" dür.

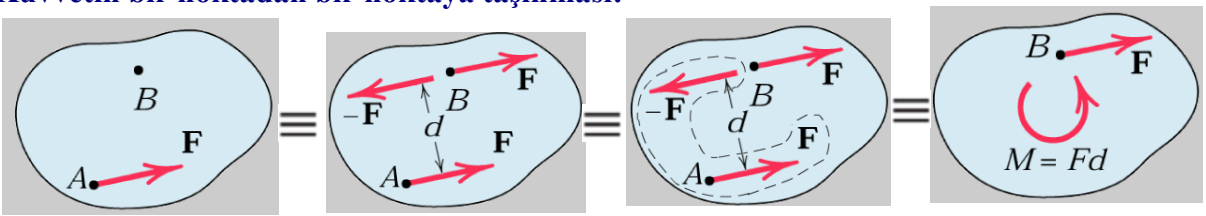


Vektörel olarak:

$$\mathbf{M}_O = \mathbf{r}_A \times \mathbf{F} + \mathbf{r}_B \times (-\mathbf{F})$$
$$= (\mathbf{r}_A - \mathbf{r}_B) \times \mathbf{F} = \mathbf{r} \times \mathbf{F}$$

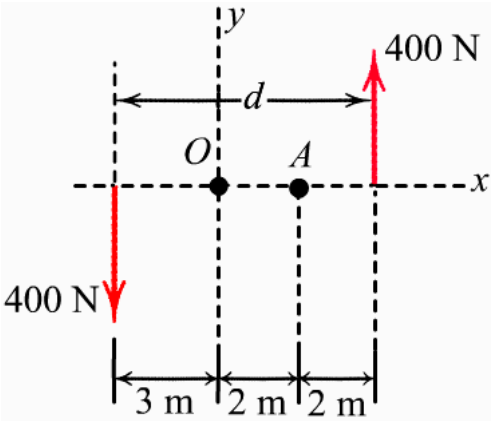


Kuvvetin bir noktadan bir noktaya taşınması:



Örnek Problem: Şekilde verilen 400 N'luk iki kuvvetin; a) O noktasına, b) A noktasına göre momentini bulunuz.

Çözüm:



Kuvvet çiftinin momenti;

$$M = Fd = 400(7) = 2800 \text{ N·m}$$

(a) $M_o = 400(3) + 400(4) = 2800$ N·m CCW

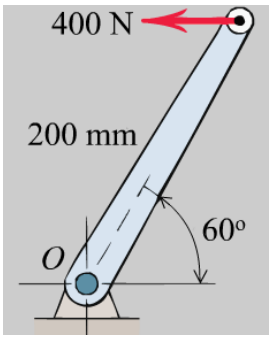
(b) $M_A = 400(5) + 400(2) = 2800$ N·m CCW

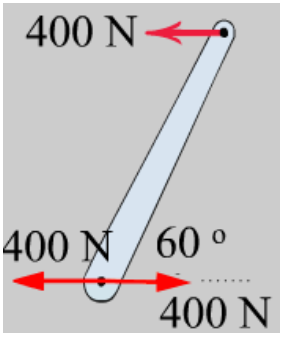
İki şıktaki sonuçlar aynı çıkmıştır. Sonuç olarak kuvvet çifti ile ilgili olan moment değerinin moment alınan noktadan bağımsız olduğunu söyleyebiliriz. Momenti,

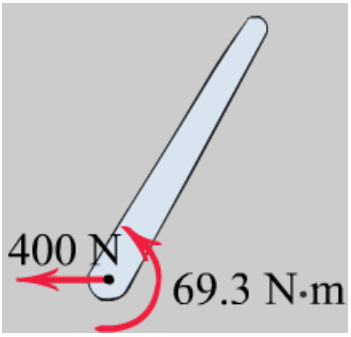
$M_o = Fd = 400(7) = 2800$ N·m **ile de bulabilirdik.**

Örnek Problem: Şekilde verilen 400 N'luk yatay kolu O noktasına (eşdeğer bir kuvvet ve moment çifti olarak) indirgeyiniz.

Çözüm:

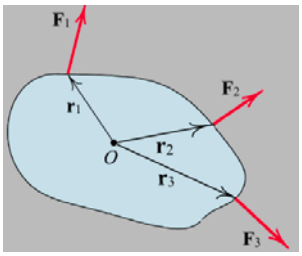


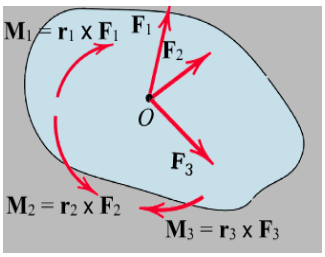




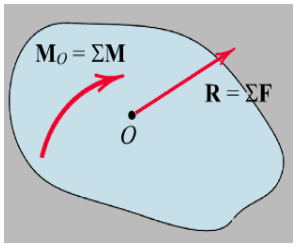
$$M = Fd = 400\,(0.200 \sin 60^\circ) = 69.3 \text{ N·m}$$

2.2.4 Kuvvetler Sisteminin İndirgenmesi

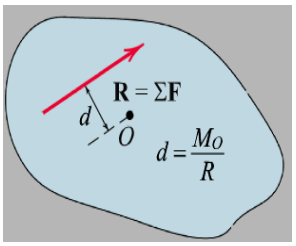




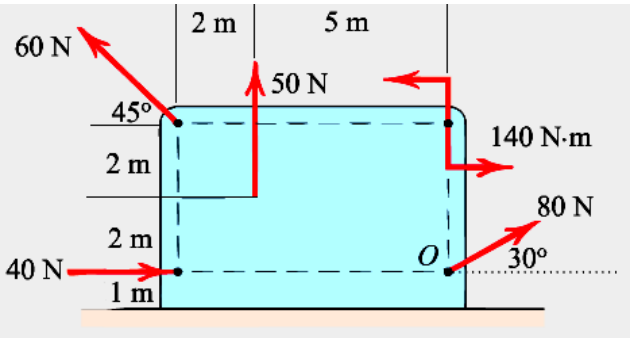
F_1, F_2 ve **F_3** ün O'ya taşınması



Eşdeğer bir kuvvet ve momente indirgeme



Kuvvet ve Momenti tek kuvvete indirgeme



Örnek Problem: Şekildeki plakaya etkiyen kuvvet çifti ve kuvvetleri eşdeğer bir kuvvete indirgeyiniz.

Çözüm:

$[R_x = \Sigma F_x]$ $\quad R_x = 40 + 80 \cos 30^o - 60 \cos 45^o = 66.9$ N

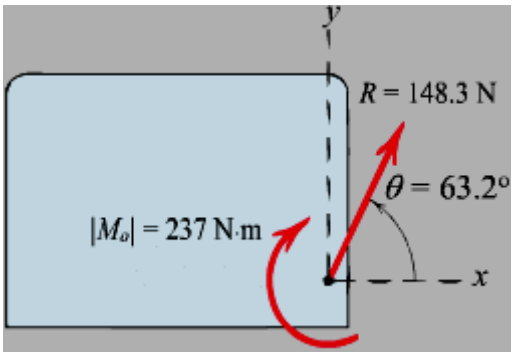
$[R_y = \Sigma F_y]$ $\quad R_y = 50 + 80 \sin 30^o + 60 \sin 45^o = 132.4$ N

$[R = \sqrt{R_x^2 + R_y^2}]$ $\quad R = \sqrt{(66.9)^2 + (132.4)^2} = 148.3$ N

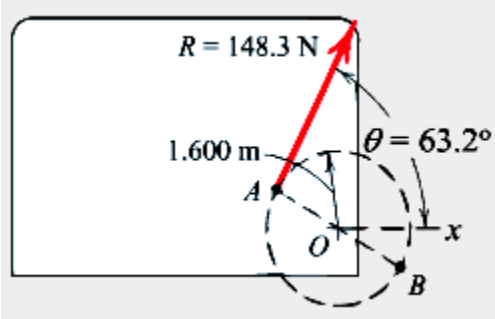
$\left[\theta = \tan^{-1} \frac{R_y}{R_x}\right]$ $\quad \theta = \tan^{-1} \frac{132.4}{66.9} = 63.2^o$

$[\circlearrowleft + \; M_o = \Sigma(Fd)]$ $\quad M_o = 140 - 50(5) + 60 \cos 45^o(4) - 60 \sin 45^o(7)$

$= -237$ N·m

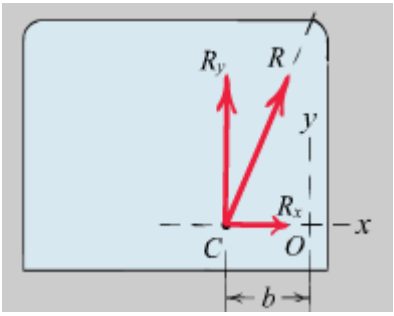


Tek kuvvete indirgeme:
Bunun için dört metot kullanılabilir:
1.



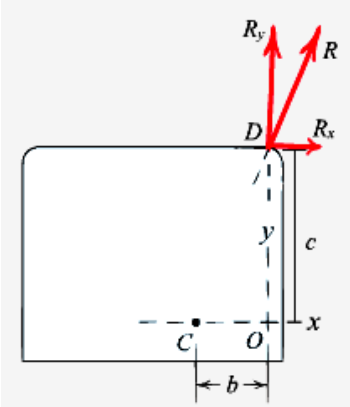
$$[Rd = |M_o|]$$
$$1.483d = 237$$
$$d = 1.600 \text{ m}$$

2.



$$R_y\, b = |M_o|$$
$$b = \frac{237}{132.4} = 1.792 \text{ m}$$

3.



$$R_x\, d = |M_o|$$
$$c = \frac{237}{66.9} = 3.54 \text{ m}$$

4.Daha genel bir yaklaşım için vektörel analiz kullanılabilir:

$$\mathbf{r} \times \mathbf{R} = \mathbf{M}_o \qquad \mathbf{r} = x\mathbf{i} + y\mathbf{j}$$

burada **r,** O'dan başlayıp **R**'nin ekti çizgisi üzerindeki herhangi bir noktada biten bir konum vektörüdür.

$$(x\mathbf{i} + y\mathbf{j}) \times (66.9\mathbf{i} + 132.4\mathbf{j}) = -237\mathbf{k}$$
$$(132.4x - 66.9y)\mathbf{k} = -237\mathbf{k}$$

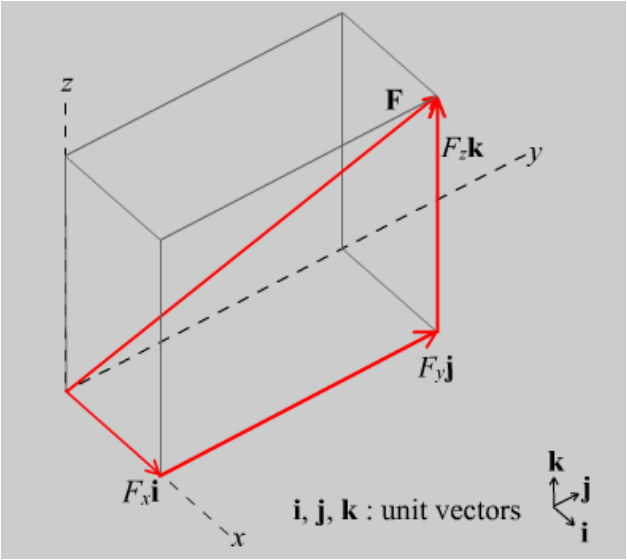
$$132.4x - 66.9y = -237$$

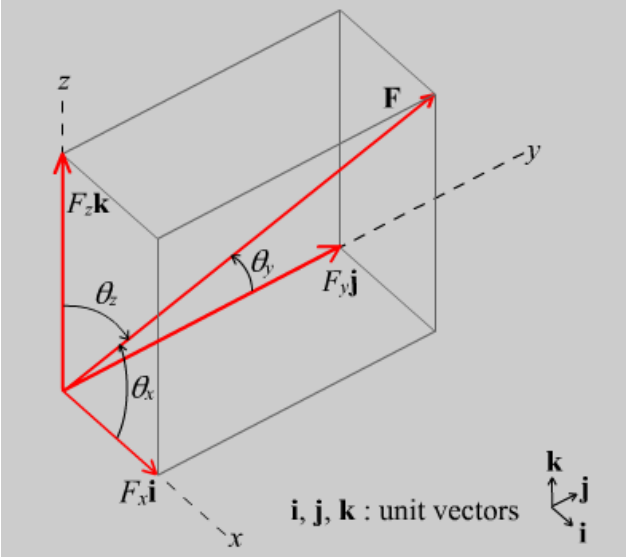
x=0 için y=3.54 m bulunur.

y=0 için x=-1.792 m bulunur. Skaler çözümle aynı sonuçlar elde edilir.

2.3 Üç Boyutlu (3-D) Kuvvet Sistemleri

2.3.1 Dik Bileşenler:





$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$$

$$\mathbf{F} = F\cos\theta_x\,\mathbf{i} + F\cos\theta_y\,\mathbf{j} + F\cos\theta_z\,\mathbf{k}$$

$$\mathbf{F} = F(\cos\theta_x\,\mathbf{i} + \cos\theta_y\,\mathbf{j} + \cos\theta_z\,\mathbf{k})$$

$$\mathbf{F} = F(l\mathbf{i} + m\mathbf{j} + n\mathbf{k})$$

$$\mathbf{F} = F(l\mathbf{i} + m\mathbf{j} + n\mathbf{k}) = F\mathbf{e}_n$$

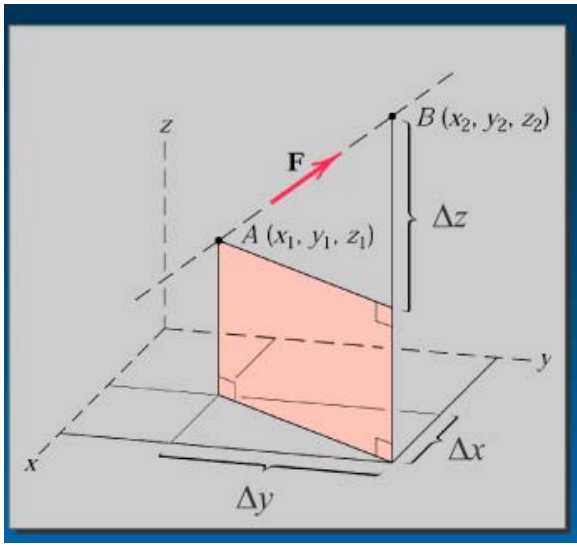
$\mathbf{e}_n$: F vektörü yönünde birim vektör

$$\left.\begin{aligned} l &= \cos\theta_x \\ m &= \cos\theta_y \\ n &= \cos\theta_z \end{aligned}\right\}$$ F vektörünün doğrultman kosinüsleri

$$l^2 + m^2 + n^2 = 1$$

$\mathbf{e_n}$ burada **F** kuvvetinin doğrultusunu gösteren birim vektördür.

$\mathbf{e_n}$'i bulmak için konum vektörlerinden de yararlanılır. Ör; herhangi iki A ve B noktası arasındaki konum vektörünü ve birim vektör şu şekilde bulunabilir.



$$\mathbf{F} = F\,\mathbf{e}_n = F\,\frac{\overrightarrow{AB}}{\overline{AB}}$$

$$= F\,\frac{\Delta x\,\mathbf{i} + \Delta y\,\mathbf{j} + \Delta z\,\mathbf{k}}{\sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}}$$

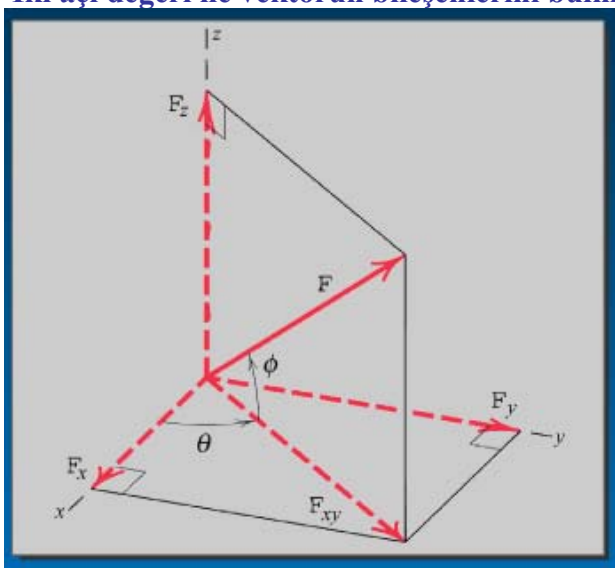
$A = A(x_1, y_1, z_1)$ $B = B(x_2, y_2, z_2)$

$\Delta x = x_2 - x_1$

$\Delta y = y_2 - y_1$

$\Delta z = z_2 - z_1$

İki açı değeri ile vektörün bileşenlerini bulma:



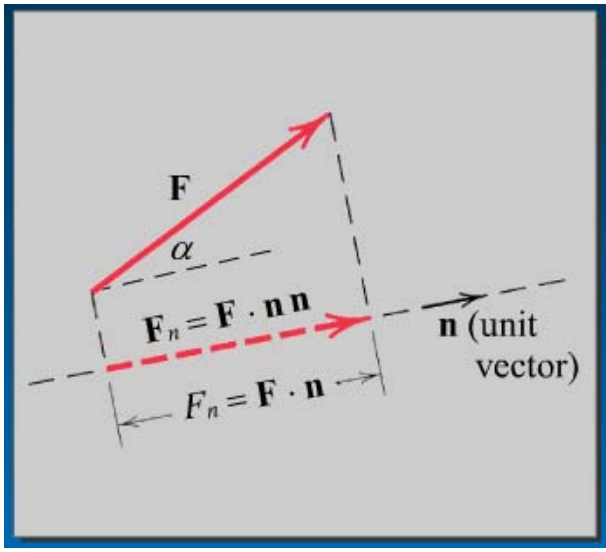
$$F_{xy} = F\cos\phi$$

$$F_z = F\sin\phi$$

$$F_x = F_{xy}\cos\theta = F\cos\phi\cos\theta$$

$$F_y = F_{xy}\sin\theta = F\cos\phi\sin\theta$$

Herhangi bir F kuvvetinin n birim vektörü üzerinde izdüşümü



n-yönünde izdüşüm uzunluğu

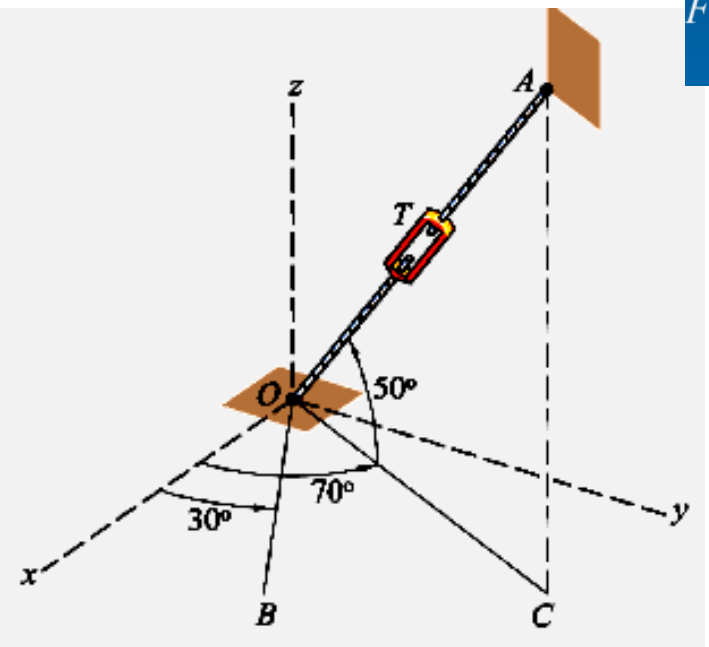
$$F_n = \mathbf{F}\cdot\mathbf{n} = F\,(1)\cos\alpha$$

izdüşüm vektörü

$$\mathbf{F}_n = F_n\,\mathbf{n} = (\mathbf{F}\cdot\mathbf{n})\,\mathbf{n} = \mathbf{F}\cdot\mathbf{n}\,\mathbf{n}$$

(Parantez gerekli değil)

Örnek Problem: T'deki sistem ile OA kablosu 8 kN kuvvete kadar gerildiğine göre; O noktasına etki eden kuvveti vektör olarak ifade ediniz. Bu kuvvetin y-ekseni ve OB çizgisi üzerindeki izdüşümünü bulunuz.



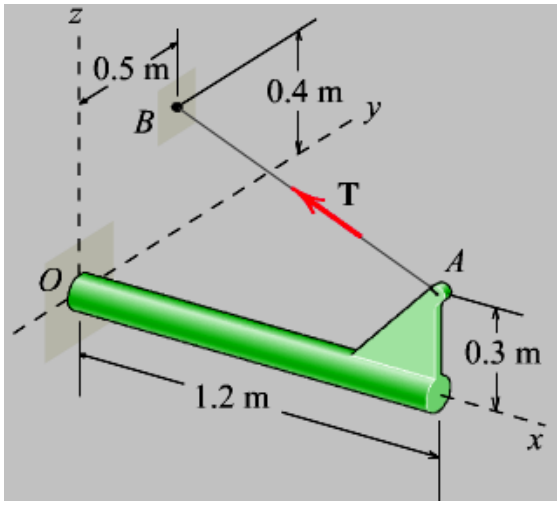
$$F = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$$
$$= 1.759\mathbf{i} + 4.83\mathbf{j} + 6.13\mathbf{k} \text{ kN}$$

$$F_y = \mathbf{F}\cdot\mathbf{j}$$
$$= (1.759\mathbf{i} + 4.83\mathbf{j} + 6.13\mathbf{k})\cdot\mathbf{j}$$
$$= 4.83 \text{ kN}$$

$$F_{OB} = \mathbf{F}\cdot\mathbf{n}_{OB}$$
$$= (1.759\mathbf{i} + 4.83\mathbf{j} + 6.13\mathbf{k})\cdot(\cos 30^o\mathbf{i} + \sin 30^o\mathbf{j})$$
$$= 1.759(\cos 30^o) + 4.83(\sin 30^o)$$
$$= 3.94 \text{ kN} \qquad \text{(as a scalar)}$$

$$\mathbf{F}_{OB} = F_{OB}\mathbf{n}_{OB}$$
$$= 3.94(\cos 30^o\mathbf{i} + \sin 30^o\mathbf{j})$$
$$= 3.41\mathbf{i} + 1.970\mathbf{j} \text{ kN}$$

Örnek Problem: Şekildeki kablo ile A noktasına 3 kN'luk bir kuvvet uygulanmıştır. **T** kuvvetini vektör olarak ifade ediniz. Bu kuvvetin OB doğrultusundaki skaler izdüşümünü bulunuz.



$$\mathbf{T} = T\mathbf{n}_{AB} \qquad \mathbf{n}_{AB} = \frac{\mathbf{AB}}{AB}$$

$$\mathbf{AB} = -1.2\mathbf{i} + 0.5\mathbf{j} + 0.1\mathbf{k} \text{ m}$$
$$AB = \sqrt{(-1.2)^2 + (0.5)^2 + (0.1)^2} \text{ m}$$
$$AB = 1.304 \text{ m}$$

$$\mathbf{n}_{AB} = \frac{\mathbf{AB}}{AB}$$
$$\mathbf{n}_{AB} = \frac{\mathbf{AB}}{AB} = \frac{-1.2\mathbf{i} + 0.5\mathbf{j} + 0.1\mathbf{k}}{1.304}$$
$$= -0.920\mathbf{i} + 0.384\mathbf{j} + 0.076\mathbf{k}$$

$$\mathbf{T} = T\,\mathbf{n}_{AB}$$
$$\mathbf{T} = 3(0.920\mathbf{i} + 0.384\mathbf{j} + 0.076\mathbf{k})$$
$$= -2.76\mathbf{i} + 1.149\mathbf{j} + 0.230\mathbf{k} \text{ kN}$$

$$T_{OB} = \mathbf{T}\cdot\mathbf{n}_{OB} = \mathbf{T}\cdot\frac{\mathbf{OB}}{OB}$$

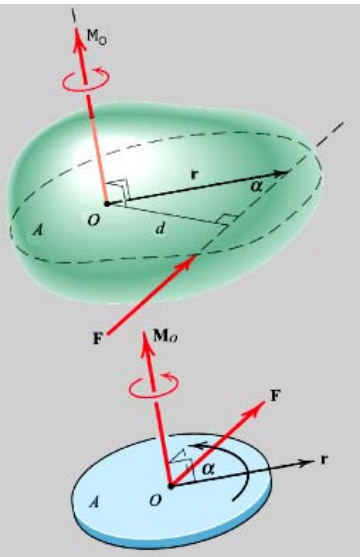
$$\mathbf{OB} = 0.5\mathbf{j} + 0.4\mathbf{k}\ \text{m}$$

$$OB = 0.640\ \text{m}$$

$$\mathbf{n}_{OB} = 0.781\mathbf{j} + 0.625\mathbf{k}$$

$$\begin{aligned} T_{OB} &= \mathbf{T}\cdot\mathbf{n}_{OB} \\ &= (-2.76\mathbf{i} + 1.149\mathbf{j} + 0.230\mathbf{k}) \\ &\qquad \cdot(0.781\mathbf{j} + 0.625\mathbf{k}) \\ &= (-2.76)(0) + (1.149)(0.781) \\ &\quad + (0.230)(0.625) \\ &= 1.041\ \text{kN} \end{aligned}$$

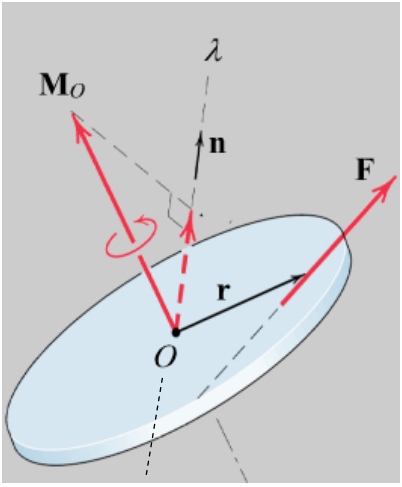
1.2.1. Moment:



$$\mathbf{M}_O = \mathbf{r}\times\mathbf{F}$$

$$M_O = rF\sin\alpha = F(r\sin\alpha) = Fd$$

Bir eksene (λ)göre moment



- Moment alınacak eksen üzerinde bir nokta seçilir, O noktası.
- Seçilen noktaya göre moment alınır: $\mathbf{M}_O = \mathbf{r}\times\mathbf{F}$
- Momentin söz konusu eksen üzerindeki izdüşümü bulunur: $M_\lambda = \mathbf{M}_O\cdot\mathbf{n}$
- Bulunan izdüşüm vektörel olarak da ifade edilebilir:

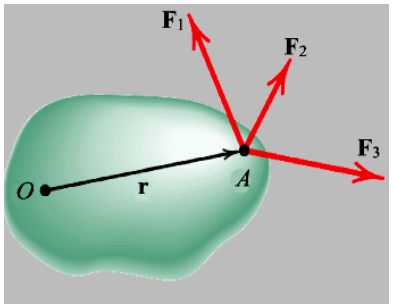
$$\mathbf{M}_\lambda = (\mathbf{M}_O\cdot\mathbf{n})\,\mathbf{n} = (\mathbf{r}\times\mathbf{F}\cdot\mathbf{n})$$

$$M_a = \mathrm{M}_O\cdot\mathrm{u}_A = \mathrm{u}_A\cdot(\mathrm{r}_A\times\mathrm{F})$$

$$M_a = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Burada $\mathbf{M_a}$ pozitif veya negatif olabilen skaler bir büyüklüktür. Pozitif işaret $\mathbf{u_A}$ birim vektörü ile $\mathbf{M_a}$ vektörünün aynı yönlü olduğunu gösterir. Negatif işaret $\mathbf{u_A}$ birim vektörü ile $\mathbf{M_a}$ vektörünün zıt yönlü olduğunu gösterir.

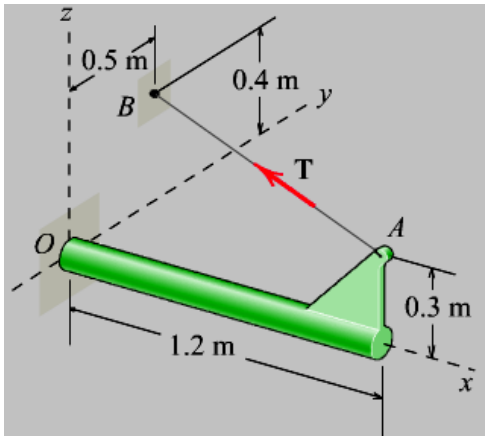
Varignon Teoremi;



$$\mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2 + \mathbf{r} \times \mathbf{F}_3 + \ldots$$
$$= \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \ldots)$$
$$= \mathbf{r} \times \Sigma \mathbf{F}$$
$$\mathbf{M}_O = \mathbf{r} \times \Sigma \mathbf{F} = \mathbf{r} \times \mathbf{R}$$

Örnek

Problem: Şekildeki kablo ile A noktasına 3 kN'luk bir kuvvet uygulanmıştır. Bu kuvvetin O noktasına ve x-eksenine göre momentini bulunuz.



$$\mathbf{F} = -2.76\mathbf{i} + 1.15\mathbf{j} + 0.230\mathbf{k} \ \text{kN}$$

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

(a) $\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F}$

$= (0.5\mathbf{j} + 0.4\mathbf{k}) \times (-2.76\mathbf{i} + 1.15\mathbf{j} + 0.230\mathbf{k})$

$= -0.345\mathbf{i} - 1.104\mathbf{j} + 1.381\mathbf{k}$ kN·m

(b) $\mathbf{M}_O = \mathbf{r}_{OB} \times \mathbf{F}$

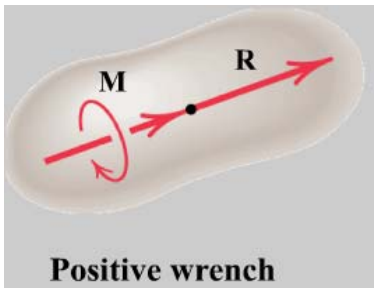
$= (1.2\mathbf{i} + 0.3\mathbf{k}) \times (-2.76\mathbf{i} + 1.15\mathbf{j} + 0.230\mathbf{k})$

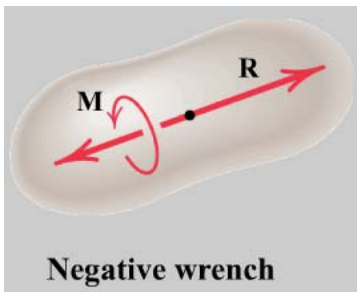
$= -0.345\mathbf{i} - 1.104\mathbf{j} + 1.381\mathbf{k}$ kN·m

$$M_x = \mathbf{M}_O \cdot \mathbf{i}$$
$$= (-0.345\mathbf{i} - 1.104\mathbf{j} + 1.381\mathbf{k}) \cdot \mathbf{i} \ \text{kN·m}$$
$$= -0.345 \ \text{kN·m}$$

"Kuvvet çifti" ve "Kuvvetler Sisteminin İndirgenmesi" konuları 2-Boyutlu kısımlarla tamamen pareleldir. O nedenle burada tekrar verilmemiştir.

1.2.2. Vida: Vida birbirlerine paralel bir kuvvet ve momentten oluşur.

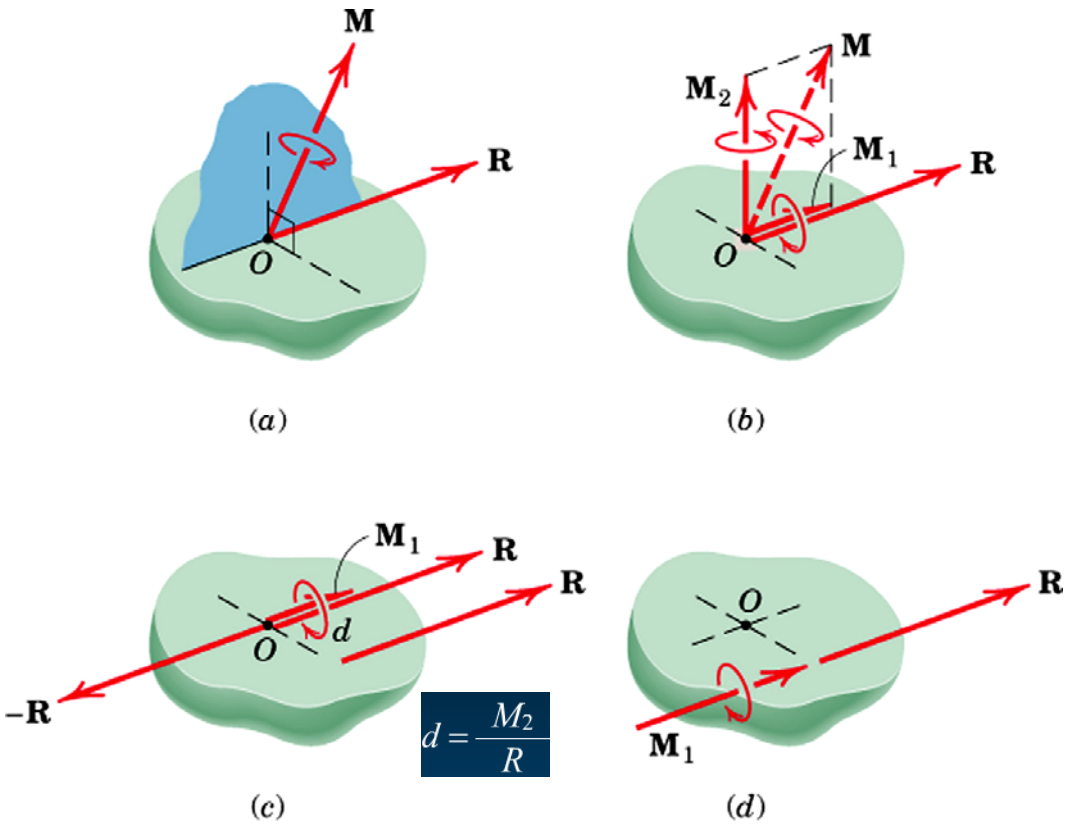




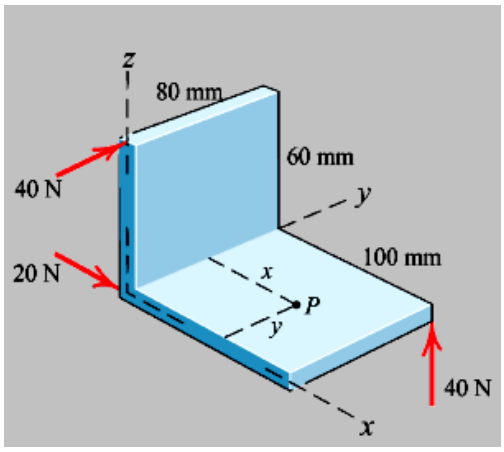
Pozitif vidaya bir tornavidanın çalışması örnek verilebilir.

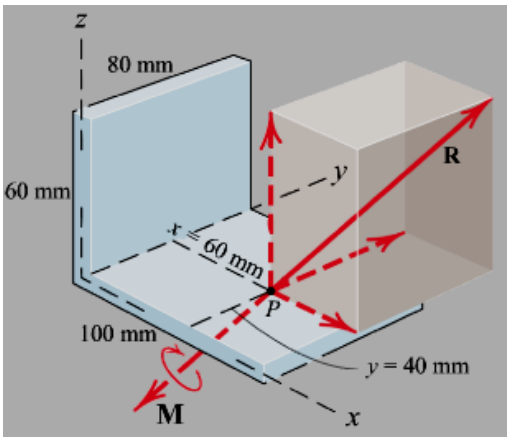
Bir kuvvetler sisteminin vidaya indirgenmesi;

a) Bir kuvvetler sistemi genel olarak birbirlerine paralel olmayan bir kuvvet ve bir momente indirgenebilir.
b) Bileşke moment bileşke kuvvete paralel ve ona dik bileşenlerine ayrılır.
c) M_2 nin yerine eşdeğer bir kuvvet çifti (**R** ve **-R**) belirlenir.
d) O noktasındaki **R** ve **–R**' dengelendiği için kuvvetler sistemi vidaya indirgenir.



Örnek Problem: Şekildeki dirseğe etki eden kuvvetleri P noktasından geçen bir vidaya indirgenebilmesi için (x,y) koordinatları ne olmalıdır. Vidayı oluşturan moment değeri nedir?





$$\mathbf{R} = \Sigma\mathbf{F} = 20\mathbf{i} + 40\mathbf{j} + 40\mathbf{k} \ \text{N}$$

$$R = \sqrt{20^2 + 40^2 + 40^2} = 60 \ \text{N}$$

$(20\mathbf{i}\ \text{N})$: $20y\ \mathbf{k}$ N·mm

$(40\mathbf{j}\ \text{N})$: $-40(60)\mathbf{i} - 40x\ \mathbf{k}$
$= -2400\mathbf{i} - 40x\ \mathbf{k}$ N·mm

$(40\mathbf{k}\ \text{N})$: $40(80-y)\ \mathbf{i} - 40(100-x)\ \mathbf{j}$
$= (3200 - 40y)\ \mathbf{i} - (4000 - 40x)\ \mathbf{j}$ N·mm

P'ye göre toplam moment:

$$\mathbf{M} = (800 - 40y)\ \mathbf{i} + (40x - 4000)\mathbf{j} + (20y - 40x)\mathbf{k} \ \text{N·mm}$$

Vidayı oluşturan kuvvet ve momentin doğrultusu aynı olacağından; **R** ve **M**'nin doğrultu kosinüslerinin eşitliğinden:

$$\cos\theta_x = \frac{R_x}{R} = \frac{20}{60} = \frac{1}{3}$$
$$\cos\theta_y = \frac{R_y}{R} = \frac{40}{60} = \frac{2}{3}$$
$$\cos\theta_z = \frac{R_z}{R} = \frac{40}{60} = \frac{2}{3}$$

$$\cos\theta_x = \frac{M_x}{M} = \frac{800 - 40y}{M}$$
$$\cos\theta_y = \frac{M_y}{M} = \frac{40x - 4000}{M}$$
$$\cos\theta_z = \frac{M_z}{M} = \frac{20y - 40x}{M}$$

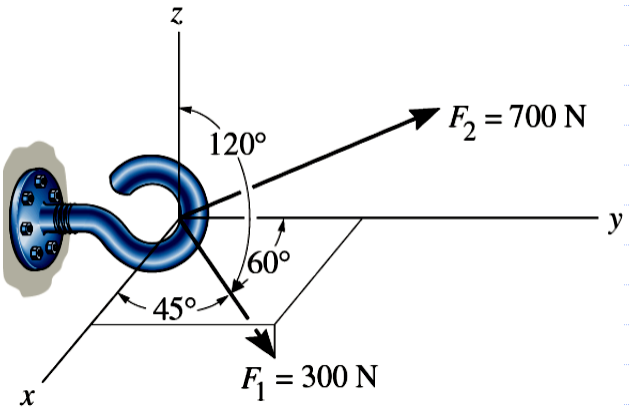
$$\frac{800 - 40y}{M} = \frac{1}{3}, \quad \frac{40x - 4000}{M} = \frac{1}{3}, \quad \frac{20y - 40x}{M} = \frac{2}{3}$$

$x = 60$ mm
$y = 40$ mm
$M = -2400$ N·mm

M'in işaretinin (-) çıkması negatif vida olduğunu gösterir.

Uygulamalar

P 2.1: Kancaya uygulanan iki kuvvetin bileşkesi +y yönünde 800 N ise $\mathbf{F_2}$ vektörünün doğrultman kosinüslerinin bulunuz.



ÇÖZÜM:

F_1 :

$$\alpha_1 = 45^o \quad \beta_1 = 60^o \quad \gamma_1 = 120^o$$

$$F_1 = F_1 \cos\alpha_1 i + F_1 \cos\beta_1 j + F_1 \cos\gamma_1 k$$

$$F_1 = (300\,N)\cos 45^o i + (300\,N)\cos 60^o j + (300\,N)\cos 120^o k$$

$$F_1 = \left[212.2i + 150j - 150k\right]N$$

$$F_2 = F_{2x} i + F_{2y} j + F_{2z} k$$

$$F_R = \left[800j\right]N$$

$$F_R = F_1 + F_2$$

$$800j = 212.2i + 150j - 150k + F_{2x} i + F_{2y} j + F_{2z1} k$$

$$800j = (212.2 + F_{2x})i + (150 + F_{2y})j + (-150 + F_{2z})k$$

$$F_{Rx} = 212.2 + F_{2x} = 0 \quad \Rightarrow \quad F_{2x} = -212.2N$$

$$F_{Ry} = 150 + F_{2y} = 800 \quad \Rightarrow \quad F_{2y} = 650N$$

$$F_{Ry} = -150 + F_{2z} = 0 \quad \Rightarrow \quad F_{2z} = 150N$$

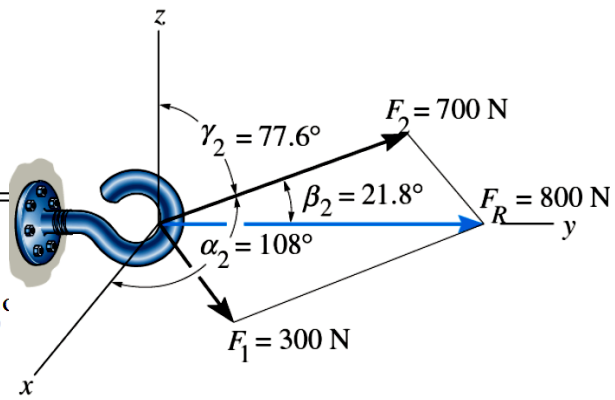
$$F_2 = \left(-212.2i + 650j + 150k\right)N$$

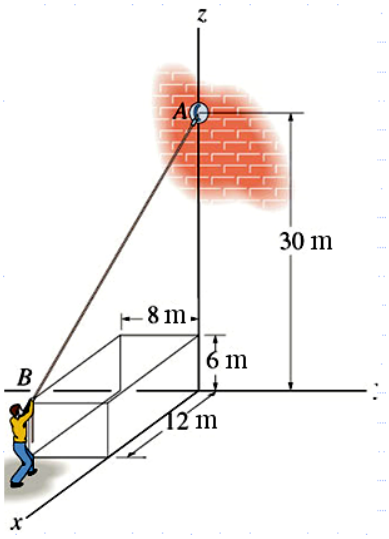
$$700 = \sqrt{(-212.2)^2 + (650)^2 + (150)^2}$$

$$\cos\alpha_2 = \frac{-212.2}{700} \quad \Rightarrow \quad \alpha_2 = \cos^{-1}\left(\frac{-212.2}{700}\right) =$$

$$\cos\beta_2 = \frac{650}{700} \quad \Rightarrow \quad \beta_2 = \cos^{-1}\left(\frac{650}{700}\right) = 21.8^o$$

$$\cos\gamma_2 = \frac{150}{700} \quad \Rightarrow \quad \gamma_2 = \cos^{-1}\left(\frac{150}{700}\right) = 77.6^o$$





P 2.2: Şekildeki çocuk ipi 70 N luk bir kuvvetle çekiyor. A noktasına etkiyen kuvveti kartezyen koordinatlarda ifade ediniz. Vektörün doğrultusunu bulunuz.

ÇÖZÜM:

Konum vektörü:

$$\mathbf{r}_{AB} = (x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}$$

$$A\,(0\,,0\,,30\,) \qquad B(12\,,-8\,,6\,)$$

$$\mathbf{r}_{AB} = (12-0)\mathbf{i} + (-8-0)\mathbf{j} + (6-30)\mathbf{k}$$

$$\mathbf{r}_{AB} = \{12\mathbf{i} - 8\mathbf{j} - 24\mathbf{k}\}\ m$$

Birim vektör:

$$\mathbf{r}_{AB} = \{12\mathbf{i} - 8\mathbf{j} - 24\mathbf{k}\}m$$

$$r_{AB} = \sqrt{(12)^2 + (-8)^2 + (-24)^2} = 28\,m$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{1}{28}(12\mathbf{i} - 8\mathbf{j} - 24\mathbf{k}) = \frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Kuvvet vektörü:

$$\mathbf{u}_{AB} = \frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{F} = F\mathbf{u}_{AB} = 70\ \mathrm{N}\left(\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)$$

$$\mathbf{F} = (30\mathbf{i} - 20\mathbf{j} - 60\mathbf{k})\ \mathrm{N}$$

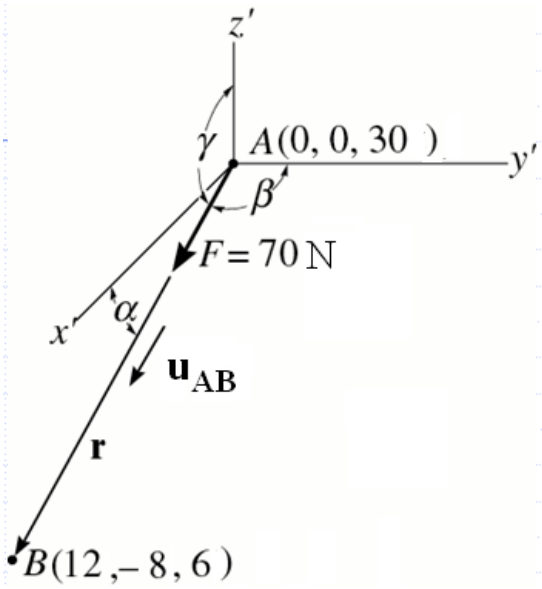
Doğrultman kosinüsleri ve açıları:

$$\mathbf{u}_{AB} = \frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

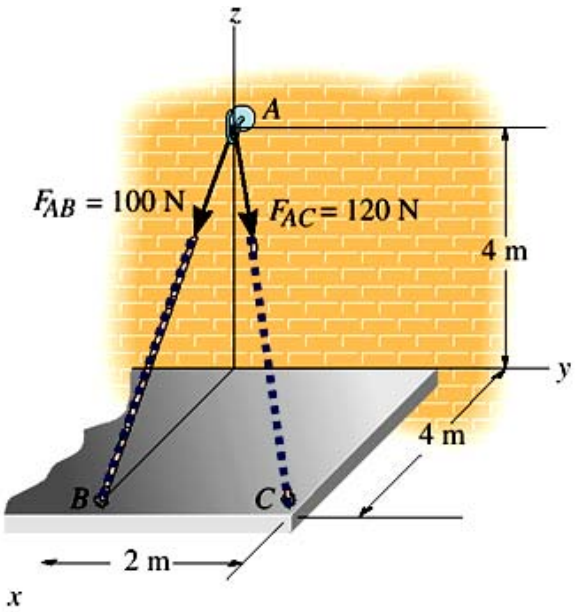
$$\cos\alpha = \frac{3}{7} \quad \Rightarrow \quad \alpha = 64.6^\circ$$

$$\cos\beta = -\frac{2}{7} \quad \Rightarrow \quad \beta = 107^\circ$$

$$\cos\gamma = -\frac{6}{7} \quad \Rightarrow \quad \gamma = 149^\circ$$



P 2.3: Şekilde görülen tavan B ve C noktalarına bağlanmış ipler yardımı ile duvarda bulunan A noktasındaki kancaya bağlanmıştır. F_{AB}=100 N ve F_{AC}=120 N olduğuna göre A nokrasına etkiyen bileşke kuvveti bulunuz.



ÇÖZÜM:

Konum vektörleri:

$$r_A = 4k \quad r_B = 4i \quad r_C = 4i + 2j$$

$$r_{AB} = r_B - r_A = (4i - 4k)m$$

$$r_{AB} = \sqrt{4^2 + (-4)^2} = 5.66m$$

$$r_{AC} = r_C - r_A = (4i + 2j - 4k)m$$

$$r_{AC} = \sqrt{4^2 + 2^2 + (-4)^2} = 6m$$

Birim vektörler:

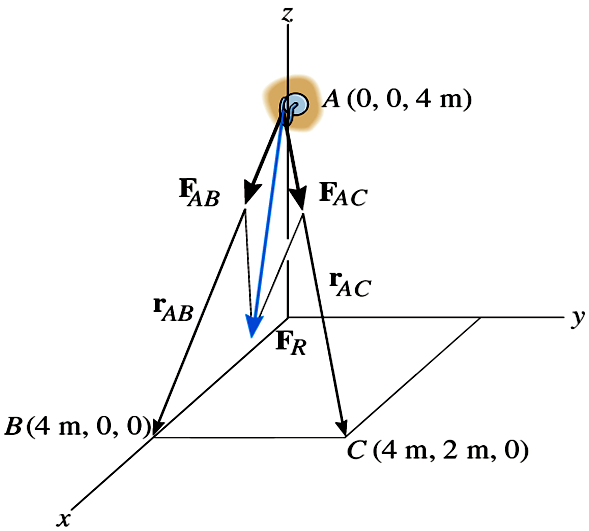
$$u_{AB} = \frac{r_{AB}}{r_{AB}} = \left(\frac{4}{5.66}i - \frac{4}{5.66}k\right)$$

$$u_{AC} = \frac{r_{AC}}{r_{AC}} = \left(\frac{4}{6}i + \frac{2}{6}j - \frac{4}{6}k\right)$$

Kuvvet Vektörleri:

$$F_{AB} = 100N(u_{AB}) = \{70.7i - 70.7k\}N$$

$$F_{AC} = 120N(u_{AC}) = \{80i + 40j - 80k\}N$$



Bileşke kuvvet:

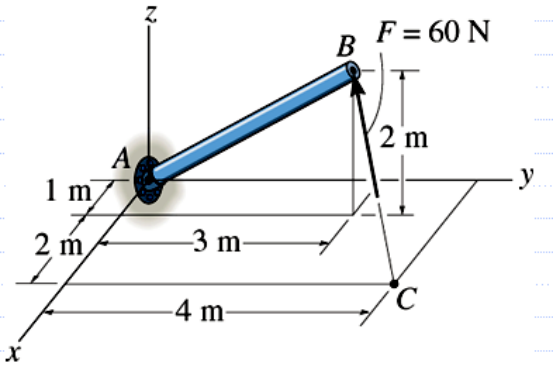
$$F_R = F_{AB} + F_{AC}$$

$$F_R = \{70.7i - 70.7k\}N + \{80i + 40j - 80k\}N$$

$$F_R = \{150.7i + 40j - 150.7k\}\ N$$

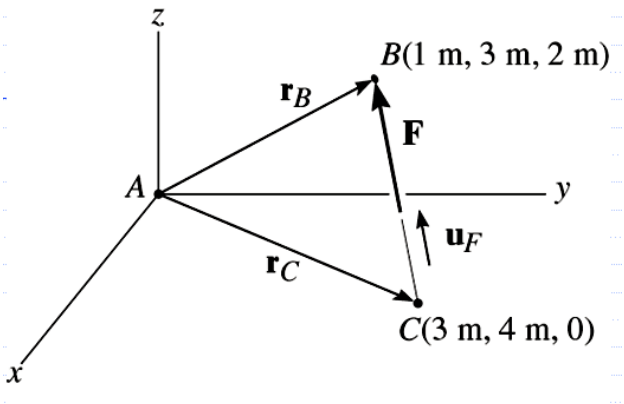
$$F_R = \sqrt{(150.7)^2 + (40)^2 + (-150.7)^2} = 217N$$

P 2.4:



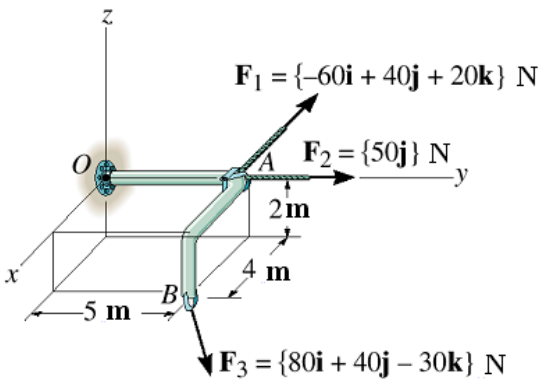
A noktasına göre momenti bulunuz.

ÇÖZÜM:

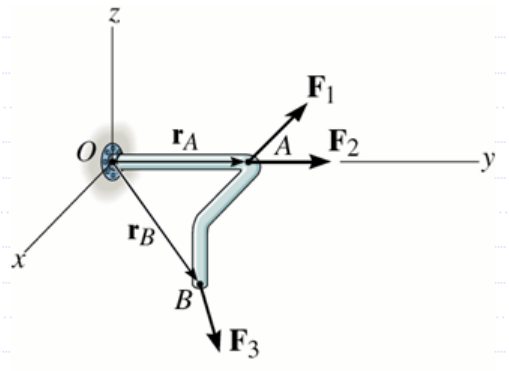


P 2.5:

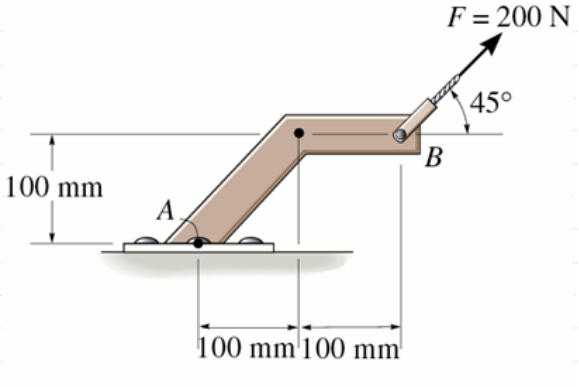
Şekilde etkiyen kuvvetler neticesinde O noktasında meydana gelen bileşke momenti ve bu bileşke momentin doğrultman kosinüslerini bulunuz.



ÇÖZÜM:

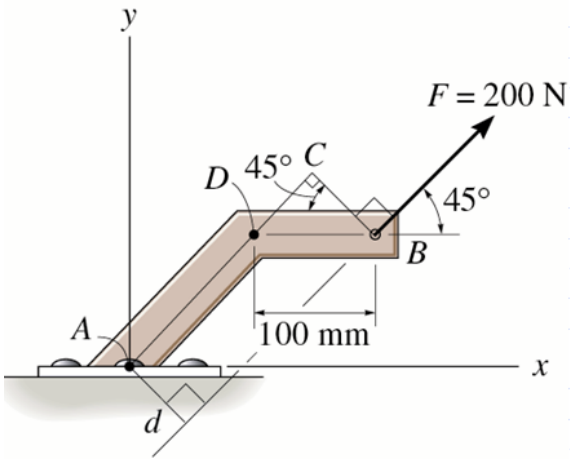


P 2.6:

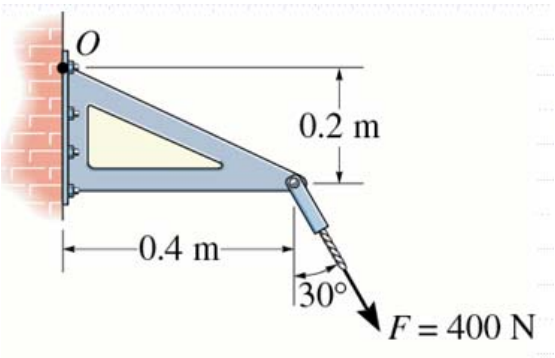


şekilde verilen kuvvetin A noktasına göre momentini bulunuz.

ÇÖZÜM:

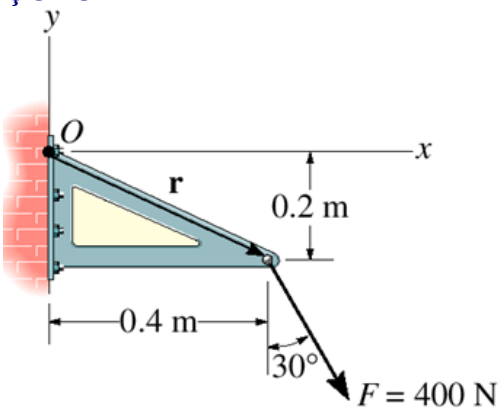


P 2.7::

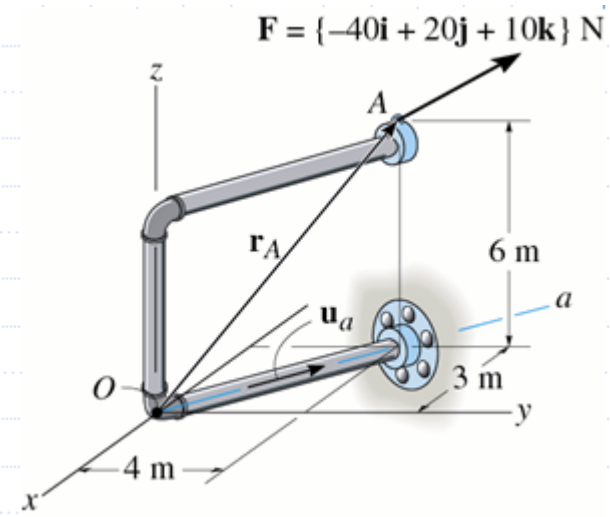


Şekilde verilen kuvvetin O noktasına göre momentini bulunuz.

ÇÖZÜM



P 2.8:



Şekildeki kuvvetin x eksenine ve OB eksenine göre momentlerini bulunuz.

ÇÖZÜM:

$$r_{OA} = \left(-3i + 4j + 6k\right)m$$

$$u_{OB} = \frac{-3i + 4j}{\sqrt{3^2 + 4^2}} = -\frac{3}{5}i + \frac{4}{5}j$$

$$M_O = r_{OA} \times F = \left(-3i + 4j + 6k\right)m \times \left(-40i + 20j + 10k\right)N$$

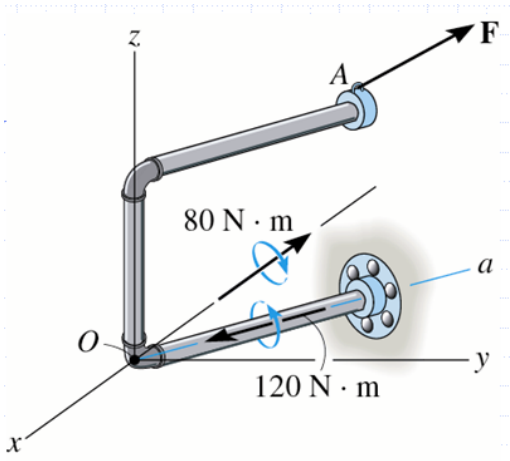
$$M_O = \left(-80i + 210j + 100k\right)N \cdot m$$

$$M_x = M_O \cdot i = \left(-80i + 210j + 100k\right)N \cdot m \cdot i = -80N \cdot m$$

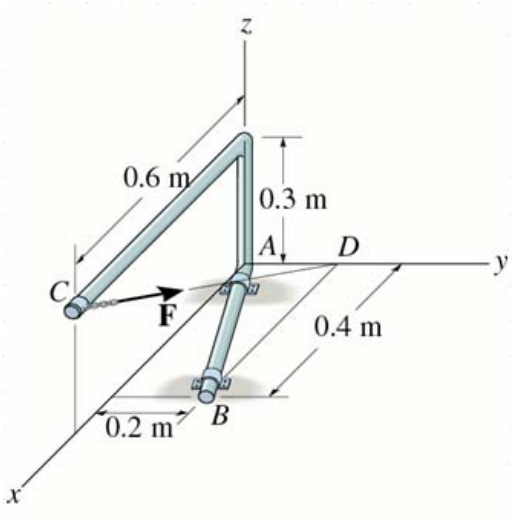
$$M_{OB} = M_O \cdot u_{OB}$$

$$= \left(-80i + 210j + 100k\right)N \cdot m \cdot \left(-\frac{3}{5}i + \frac{4}{5}j\right)$$

$$= \left(-48 + 168\right)N \cdot m = 120\,N \cdot m$$

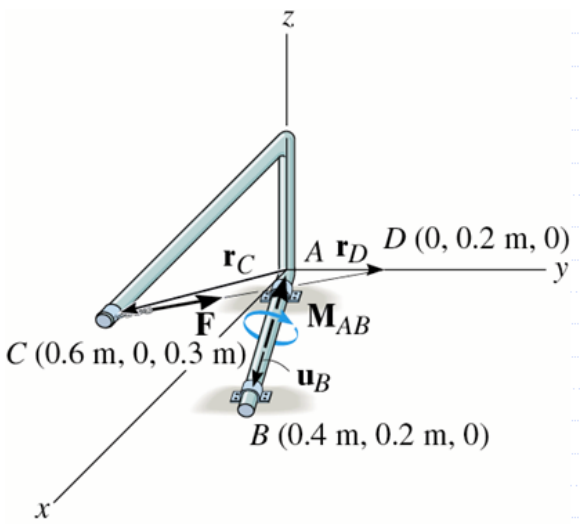


P 2.9:

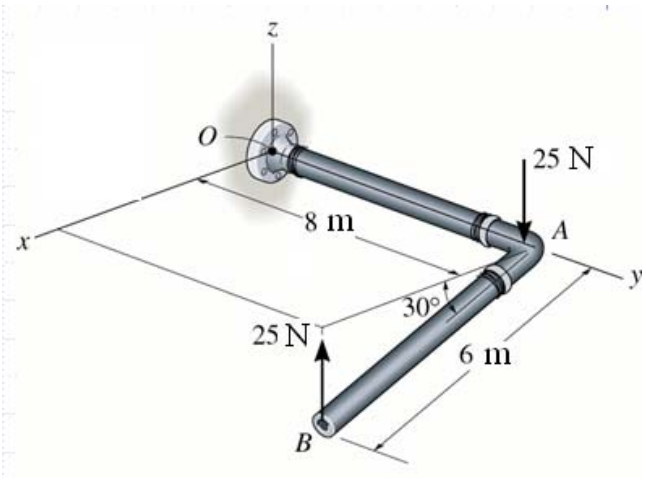


F=-600i+200j-300k kuvvetinin AB eksenine göre momentini bulunuz.

ÇÖZÜM:



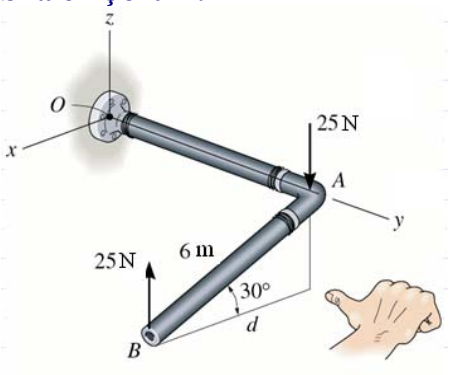
P 2.10:



Kupl momenti skaler ve vektörel olarak bulunuz.

ÇÖZÜM:

Skaler Çözüm:



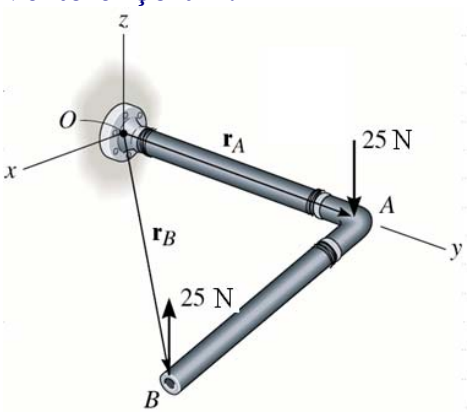
$$F = 25\ \text{N}$$

$$d = 6\cos 30^\circ = 5.2\ \text{m}$$

$$M = Fd = (25\,\text{m})(5.2\,\text{m})$$

$$M = 129.9\ \text{Nm}$$

Vektörel Çözüm:



1.

$$M_O = r_A \times (-25k) + r_B \times (+25k)$$

$$M_O = 8j \times (-25k)$$

$$+ (6\cos 30^\circ i + 8j - 6\sin 30^\circ k) \times (+25k)$$

$$M = -200i - 129.9j + 200i = (-129.9j)\ \text{M.m}$$

2.

$$M_O = r_{AB} \times (25k)$$

$$M_O = (6\cos 30^\circ i - 6\sin 30^\circ k) \times (+25k)$$

$$= (-130j)\ \text{Nm}$$