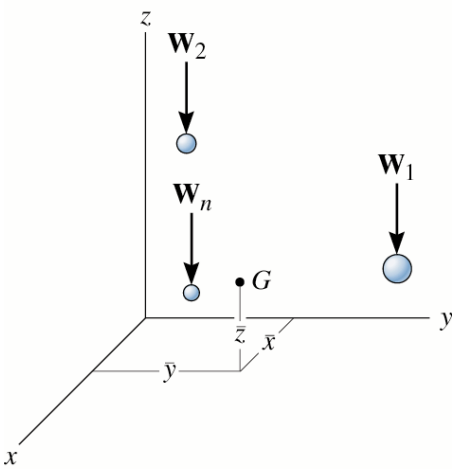
BÖLÜM 5
AĞIRLIK MERKEZİ-GEOMETRİK MERKEZ

5.1 Ağırlık Merkezi

Tanım: Ağırlık merkezi G, parçacıklar sisteminde ağırlığın bileşkesinin olduğu noktadır. Parçacıların ağırlıklarının paralel kuvvet sistemleri olduğu düşünülür. Ağırlıklar sistemi ağırlık sistemine konacak tek bir ağırlıkla değiştirilebilir.



Toplam Ağırlık:

$$W_R = \sum_{i=1}^{n} W_i$$

Ağırlık Merkezi:

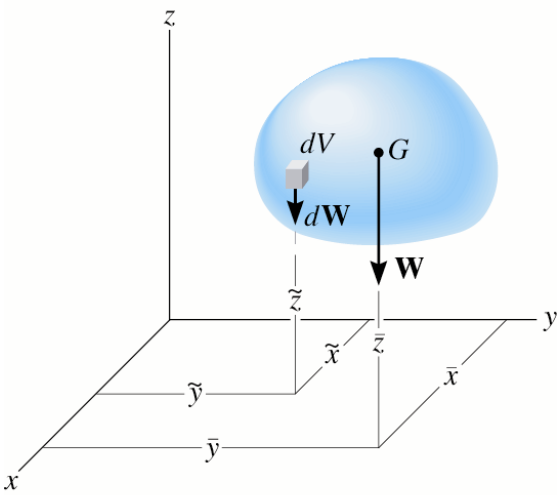
$$\bar{x} = \frac{\sum_{i=1}^{n} \tilde{x}_i W_i}{\sum_{i=1}^{n} W_i} \qquad \bar{y} = \frac{\sum_{i=1}^{n} \tilde{y}_i W_i}{\sum_{i=1}^{n} W_i} \qquad \bar{z} = \frac{\sum_{i=1}^{n} \tilde{z}_i W_i}{\sum_{i=1}^{n} W_i}$$

$\bar{x}, \bar{y}, \bar{z}$ *ağırlık merkezinin koordinatları*

$\tilde{x}_i, \tilde{y}_i, \tilde{z}_i$ *i.parçacığın koordinatı*

W_i *i.parçacığın ağırlığı*

Eğer bir yapı sonsuz sayıda partikülden oluşuyorsa ağırlık merkezine integral ifadeleri katılır.



$$\bar{x} = \frac{\int \tilde{x}\, dW}{\int dW} \qquad \bar{y} = \frac{\int \tilde{y}\, dW}{\int dW} \qquad \bar{z} = \frac{\int \tilde{z}\, dW}{\int dW}$$

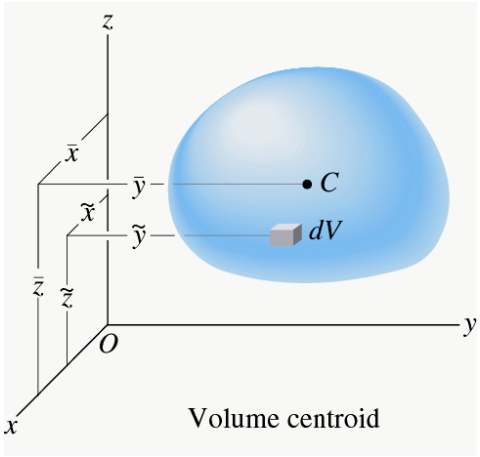
γ – *yoğunluk*

$dW = \gamma\, dV$

$$\bar{x} = \frac{\int_V \tilde{x}\, \gamma\, dV}{\int_V \gamma\, dV} \qquad \bar{y} = \frac{\int_V \tilde{y}\, \gamma\, dV}{\int_V \gamma\, dV} \qquad \bar{z} = \frac{\int_V \tilde{z}\, \gamma\, dV}{\int_V \gamma\, dV}$$

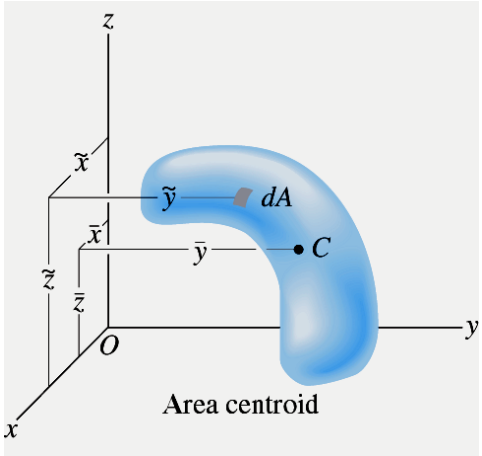
5.2 Geometrik Merkez

Tanım: Bir nesnenin geometrik merkezidir. Formülasyonu ağırlık merkezine benzer.



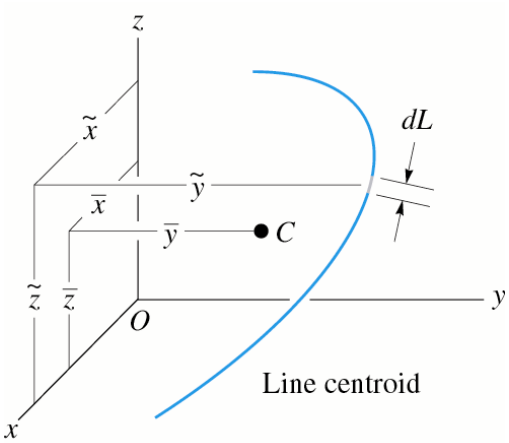
Hacimsel merkez:

$$\bar{x} = \frac{\int_V \tilde{x}\, dV}{\int_V dV} \qquad \bar{y} = \frac{\int_V \tilde{y}\, dV}{\int_V dV} \qquad \bar{z} = \frac{\int_V \tilde{z}\, dV}{\int_V dV}$$



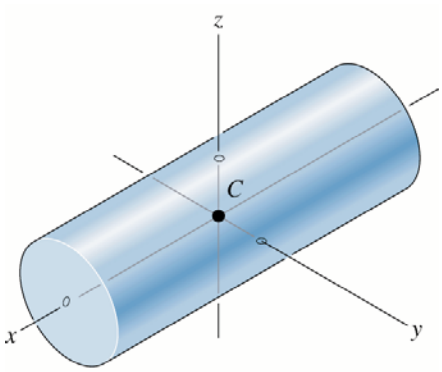
Alansal merkez:

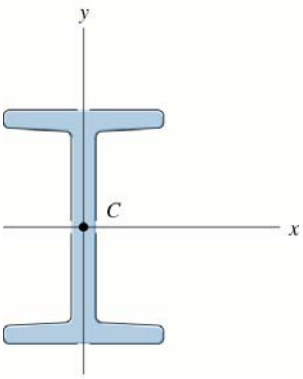
$$\bar{x} = \frac{\int_A \tilde{x}\, dA}{\int_A dA} \qquad \bar{y} = \frac{\int_A \tilde{y}\, dA}{\int_A dA} \qquad \bar{z} = \frac{\int_A \tilde{z}\, dA}{\int_A dA}$$

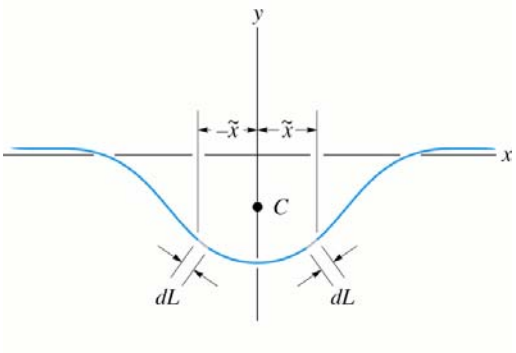


Çizgisel merkez:

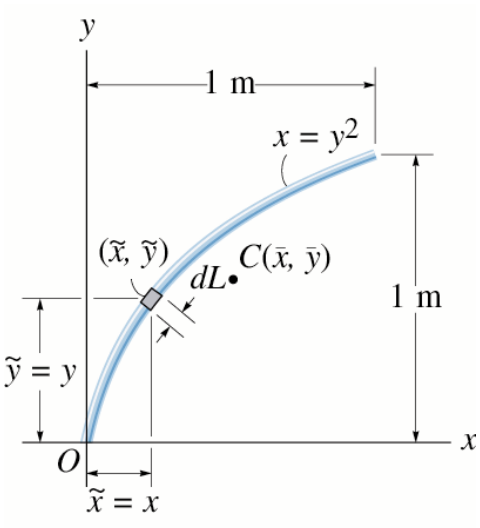
$$\bar{x} = \frac{\int_L \tilde{x}\, dL}{\int_L dL} \qquad \bar{y} = \frac{\int_L \tilde{y}\, dL}{\int_L dL} \qquad \bar{z} = \frac{\int_L \tilde{z}\, dL}{\int_L dL}$$







Örnek Problem 5.1.



Yandaki yayın geometrik merkezini bulunuz.

ÇÖZÜM:

$$dL = \sqrt{(dx)^2 + (dy)^2}$$

$$dL = \left(\sqrt{\left(\frac{dx}{dy}\right)^2 + 1}\right) dy$$

$$x = y^2$$

$$\frac{dx}{dy} = 2y$$

$$dL = \left(\sqrt{(2y)^2 + 1}\right) dy$$

$$\bar{x} = \frac{\int_L \tilde{x} dL}{\int_L dL} = \frac{\int_0^1 x\left(\sqrt{(2y)^2+1}\right) dy}{\int_0^1 \left(\sqrt{(2y)^2+1}\right) dy}$$

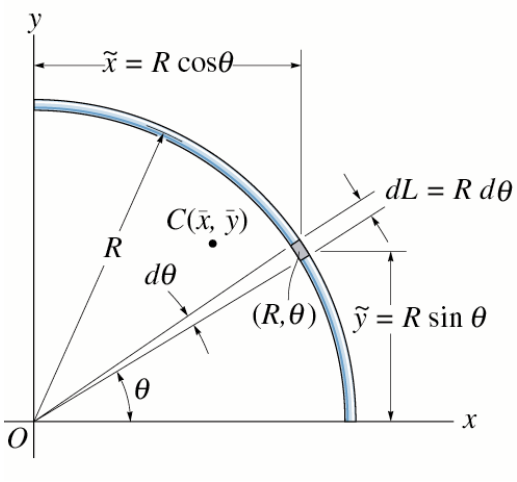
$$x = y^2$$

$$\bar{x} = \frac{\int_0^1 y^2\left(\sqrt{4y^2+1}\right) dy}{\int_0^1 \left(\sqrt{4y^2+1}\right) dy} = \frac{0.6063}{1.479} = 0.410\ m$$

$$\bar{y} = \frac{\int_L \tilde{y}\, dL}{\int_L dL} = \frac{\int_0^1 y\left(\sqrt{(2y)^2+1}\right) dy}{\int_0^1 \left(\sqrt{(2y)^2+1}\right) dy}$$

$$\bar{y} = \frac{\int_0^1 y\left(\sqrt{4y^2+1}\right) dy}{\int_0^1 \left(\sqrt{4y^2+1}\right) dy} = \frac{0.8484}{1.479} = 0.574\ m$$

Örnek Problem 5.2



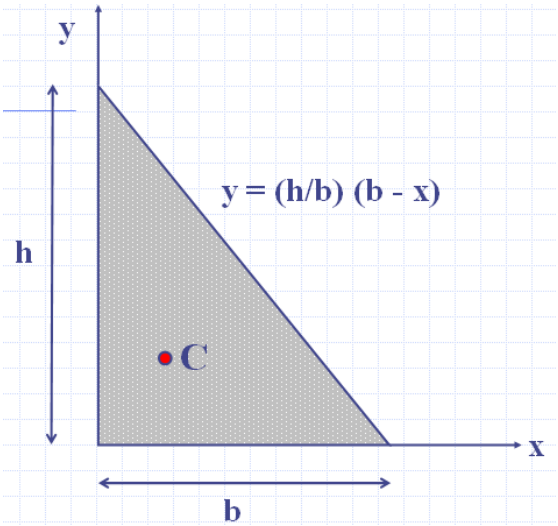
Çeyrek yayın geometrik merkezini bulunuz.

ÇÖZÜM:

$$\bar{x} = \frac{\int_L \tilde{x}\,dL}{\int_L dL} = \frac{\int_0^{\frac{\pi}{2}} (R\cos\theta)\,R\,d\theta}{\int_0^{\frac{\pi}{2}} R\,d\theta} = \frac{R^2\int_0^{\frac{\pi}{2}} \cos\theta\,d\theta}{R\int_0^{\frac{\pi}{2}} d\theta} = \frac{2R}{\pi}$$

$$\bar{y} = \frac{\int_L \tilde{y}\,dL}{\int_L dL} = \frac{\int_0^{\frac{\pi}{2}} (R\sin\theta)\,R\,d\theta}{\int_0^{\frac{\pi}{2}} R\,d\theta} = \frac{R^2\int_0^{\frac{\pi}{2}} \sin\theta\,d\theta}{R\int_0^{\frac{\pi}{2}} d\theta} = \frac{2R}{\pi}$$

Örnek Problem 5.3

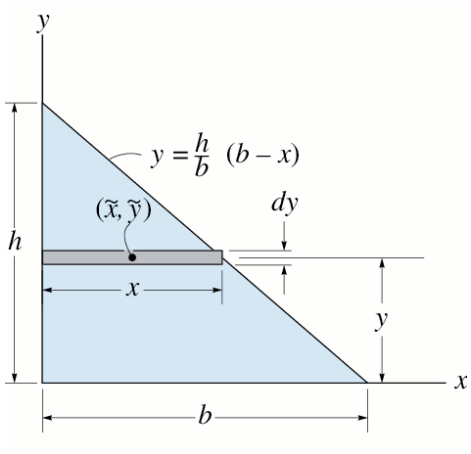


Üçgenin geometrik merkezini bulunuz.

ÇÖZÜM:

Üçgenin geometrik merkezini iki metodla bulabiliriz.

1. Şerit metodu:



$$dA = x\,dy$$

$$dA = \frac{b}{h}(h-y)\,dy$$

$$\tilde{x} = \frac{1}{2}\left(\frac{b}{h}(h-y)\right)$$

$$\tilde{y} = y$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^h y\left(\frac{b}{h}(h-y)\right)dy}{\int_0^h \left(\frac{b}{h}(h-y)\right)dy}$$

$$\bar{y} = \frac{\frac{1}{6}bh^2}{\frac{1}{2}bh} = \frac{h}{3}$$

$$\bar{x} = \frac{\int_A \tilde{x}\ dA}{\int_A dA} = \frac{\int_0^h \frac{1}{2}\left(\frac{b}{h}(h-y)\right)\left(\frac{b}{h}(h-y)\right) dy}{\int_0^h \left(\frac{b}{h}(h-y)\right) dy}$$

$$\bar{x} = \frac{\frac{1}{6}\ b^2\ h}{\frac{1}{2}\ b\ h} = \frac{b}{3}$$

2. Çift integral metodu:

$$dA = dx\,dy$$

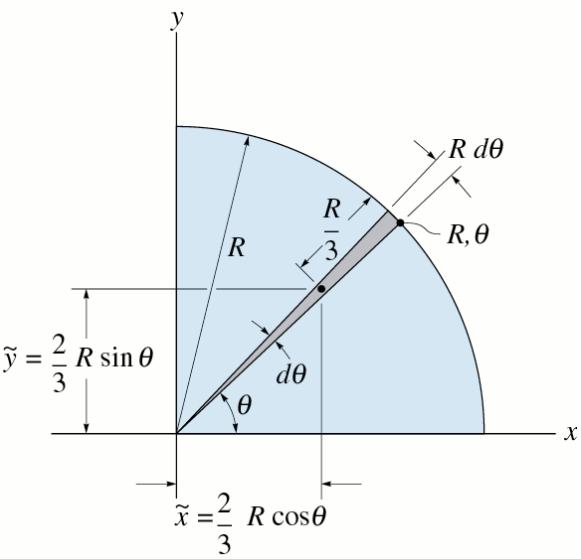
$$\tilde{x} = x$$

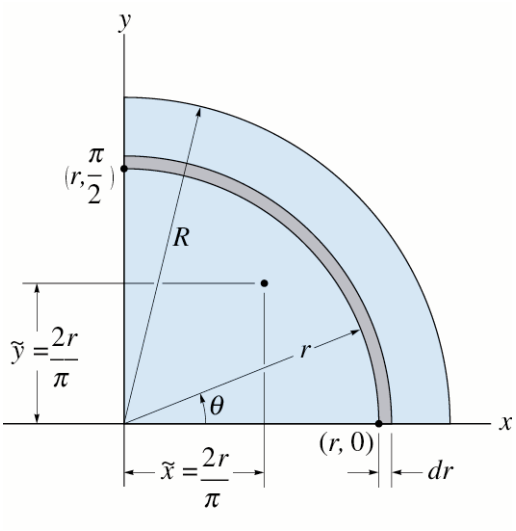
$$\tilde{y} = y$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^b \int_0^{\frac{h}{b}(b-x)} y\,dy\,dx}{\int_0^b \int_0^{\frac{h}{b}(b-x)} dy\,dx} = \frac{\int_0^b h^2/2b^2\,(b-x)^2\,dx}{\frac{1}{2}bh}$$

$$\bar{y} = \frac{\int_0^b h^2/2b^2\,(b-x)^2\,dx}{\frac{1}{2}bh} = \frac{-h^2/6b^2\,(b-x)^3}{\frac{1}{2}bh}$$

$$\bar{y} = \frac{\frac{1}{6}bh^2}{\frac{1}{2}bh} = \frac{h}{3}$$


y
R dθ
R/3
R, θ
R
ỹ = 2/3 R sin θ
dθ
θ
x
x̃ = 2/3 R cos θ


y
(r, π/2)
R
ỹ = 2r/π
r
θ
x
(r, 0)
dr
x̃ = 2r/π

Kompozit yapıların geometrik merkezi:

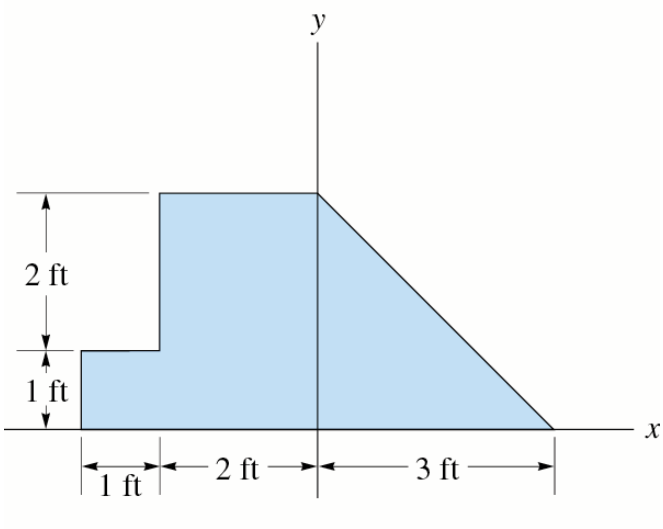
Basit yapıların birleşmesinden oluşmuş karmaşık yapılara kompozit yapılar denir. Bunların geometrik merkezi bulunurken basit yapıların geometrik merkez özelliklerinden yararlanılır.

Çözüm Yöntemi:

- Karmaşık yapı basit geometrili alt parçalara ayrılır.
- Eğer delik veya kesilmiş kısım varsa bunlar negatif alan gibi düşünülür.
- Simetri varsa geometrik merkez bu simetri ekseni üzerindedir.
- Tablo oluşturulur ve çözüm yapılır.

$$\overline{x} = \frac{\sum_{i=1}^{n} \tilde{x}_i A_i}{\sum_{i=1}^{n} A_i} \qquad \overline{y} = \frac{\sum_{i=1}^{n} \tilde{y}_i A_i}{\sum_{i=1}^{n} A_i} \qquad \overline{z} = \frac{\sum_{i=1}^{n} \tilde{z}_i A_i}{\sum_{i=1}^{n} A_i}$$

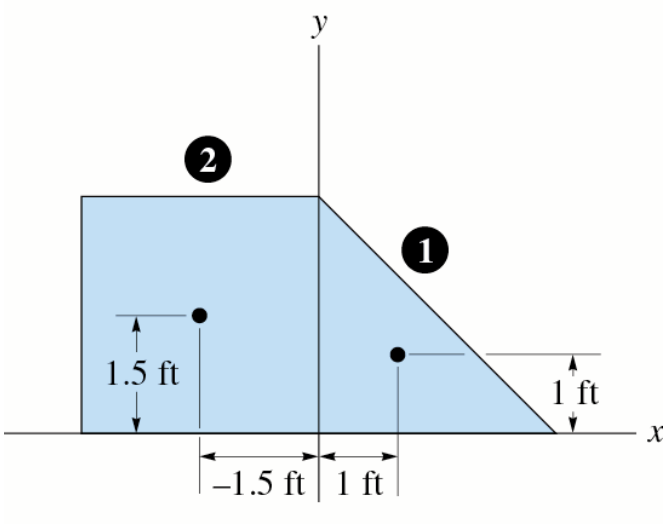
Örnek Problem 5.4

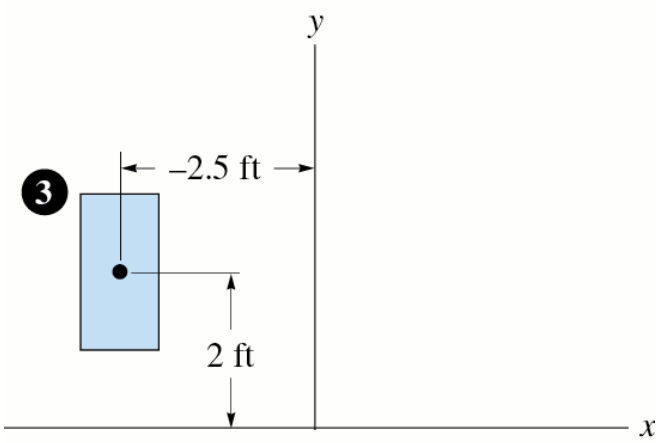


Yandaki yapının geometrik merkezini bulunuz.

ÇÖZÜM:

1 ve 2 nolu alanın toplamından 3 nolu alanın çıkarılması ile yandaki şekli elde edebiliriz.



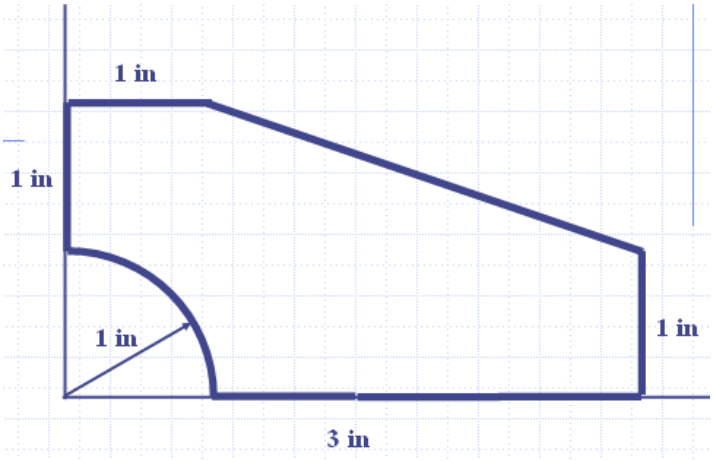


Segment	A (ft²)	x	y	xA	yA
1	4.5	1	1	4.5	4.5
2	6	-1	1.5	-6	9
3	1	-2.5	0.5	-2.5	0.5
	Σ A = 11.5			Σ xA = -4	Σ xA = 14

$$\bar{x} = \frac{\sum \tilde{x}A}{\sum A} = \frac{-4}{11.5} = -0.348\ ft$$

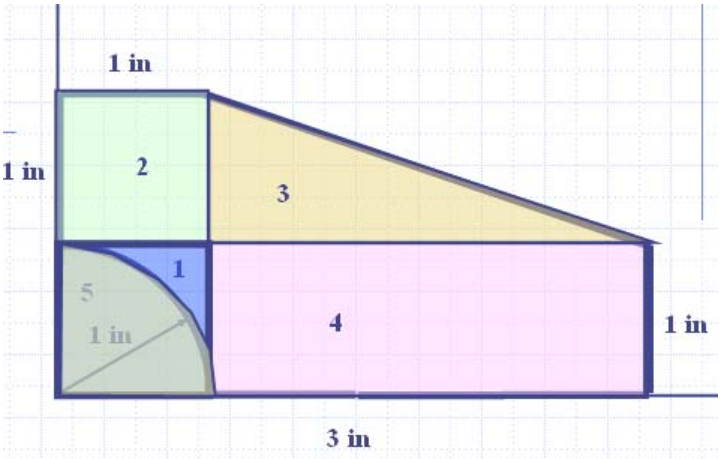
$$\bar{y} = \frac{\sum \tilde{y}A}{\sum A} = \frac{14}{11.5} = 1.22\ ft$$

Örnek Problem 5.5



Yandaki yapının geometrik merkezini bulunuz.

ÇÖZÜM



1, 2, 3, 4 alanlarından 5 alanının çıkarılması ile yandaki şekli elde edebiliriz.

Bölge	Alan	x	y	xA	yA
1	1,00000	0,50000	0,50000	0,50000	0,50000
2	1,00000	0,50000	1,50000	0,50000	1,50000
3	1,50000	2,00000	1,33333	3,00000	2,00000
4	3,00000	2,50000	0,50000	7,50000	1,50000
5	-0,78540	0,42441	0,42441	-0,33333	-0,33333
	5,71460			11,16667	5,16667
	x=	1,95406			
	y=	0,90412			